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The Relation of Microscope Size to Sieve Size for Ground Coals

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INTRODUCTION

The weight versus size distribution of a ground coal is an important parameter in many industrial processes, and there has been much study of such distributions (1). The simplest and most commonly used method of size analysis is sieve analysis, but the finest sieve which can be used with any degree of accuracy has a nominal aperture of 44 microns (U.S. standard sieve No. 325). For material below this size it is necessary to use less direct methods such as air elutriation, sedimentation velocities, and microscopic measurement (2). In the sub-sieve range down to 1 micron, measurement with an optical microscope is simple, avoids problems of agglomeration found with other methods, and gives results which are fairly reproducible. Although microscope counting and measurement is tedious, if two experienced personnel work together, results can be obtained at least as quickly as with most other methods.

To extend a sieve size distribution to sub-sieve sizes, it is necessary to know the relation between sieve size and the particular size property measured in the sub-sieve range. In the case of microscopic measurement, the size property is some characteristic visual dimension. The most commonly used dimension is the "projected area diameter", that is, the diameter of the circle which has the same area as the projected outline of the particle in the plane of the microscope slide. In the work described in this paper, the relation between microscope diameter (defined as above) and sieve size is investigated.

REVIEW OF PREVIOUS WORK

Skinner, Boas-Traube, Brown, and Hawksley (3) measured the ratio of microscopic diameter to sieve size for coal which just passed a 7-mesh sieve. They obtained a mean ratio of 1.42. They state that it is not known whether this ratio varies with size. Heywood (4) investigated this ratio for a number of different materials and found values between 1 and 1.8, depending on the geometric shape of the particles. He has also proposed (5) an empirical formula which gives this ratio as a function of m and n, where m is the microscopic breadth of the particle divided by its thickness and n is the breadth divided by the length. Heywood states (6) that the relation between microscope diameter and sieve size is dependent only on the geometry of the particles and not on their size. Guruswamy and co-workers (7) measured the ratio of microscope diameter to sieve size for a series of sieve fractions of a coal which had been broken upon being dropped on to a metal plate. The ratio appeared to increase slightly with decrease in size over a range of approximately 3000 to 100 microns sieve size, the ratio having a mean value of about 1.5.

APPARATUS AND EXPERIMENTAL TECHNIQUE

Microscopic size counts were performed on a number of sieve fractions of coal in the range of 30 to 325 U.S. standard sieve sizes. The measurements were made by projecting the field on to a ground glass screen on which circles of varying radii were drawn; the field was then moved to bring each particle under the appropriate diameter circle. By calibration with a microscopic scale, the representative size of the circles for a given magnification was known. Each particle was assigned on an area basis to a group lying between two circles. From such a count, the cumulative percentage number of particles below any given microscope size was obtained.

For sieve fractions below 170 mesh, slides were prepared in the following manner. A sample of the coal was stirred vigorously in several ml. of toluene until well dispersed, and a drop of the suspension was transferred to 1 ml. of a 10% ethyl cellulose in toluene solution. After stirring, a drop of this suspension was spread on a slide and allowed to thicken. Using this technique, extremely uniform and well dispersed fields were obtained. For larger sizes, it was found that the particles tended to project from the dried cellulose layer; and it was not possible to get clear images of such particles. Consequently, dry slides were prepared by tipping a small amount of the sieve fraction on to the slide and spreading the particles with a fine brush. For these sizes, agglomeration did not occur to any significant extent.

The coals tested were ground according to the standard Hardgrove test (8), as described in a previous paper (9).

THEORY

Let the external geometric surface area and the volume of the particle of sieve size μ be given by

$$S = k_1 \mu^2$$

and

$$V = k_2 \mu^3 .$$

Then

$$dS = k_1 \mu^2 dN$$

and

$$dV = k_2 \mu^3 dN$$

where dN represents the number of particles of size $\mu + d\mu$. Over a short size range, k_1 and k_2 may be assumed to be constant and

$$\frac{S}{V} = \frac{k_1}{k_2} \frac{\int \mu^2 dN}{\int \mu^3 dN} .$$

Now $S/V = S_o$, the specific surface area of the material; and if a shape factor k is defined by $k = S_o \mu$, then $k = k_1/k_2$. Therefore,

$$S_o = k \frac{\int \mu^2 dN}{\int \mu^3 dN} .$$

Defining R by $R = d_p/\mu$, where d_p is the projected area diameter,

$$R = \frac{S_o}{k} \frac{\int_0^N d_p^3 dN}{\int_0^N d_p^2 dN} . \qquad (1)$$

For the size fractions and coals investigated, S_o and k were known from previous determinations (9). The integrals were evaluated graphically.

Although Equation (1) is derived without recourse to the concept of mean values, k/S_o is a specific-area (geometric area per unit volume) mean-sieve size, while $\int d_p^3 dN / \int d_p^2 dN$ is a specific-area-mean-microscope size. Thus, Equation (1) can be given as

$$R = \frac{\tilde{d}_p}{\tilde{\mu}} \qquad (1a)$$

where $\tilde{\mu}$ is a specific-area-mean-sieve size and $\tilde{d}_p$ is specific-area-mean-microscope size. In general, it was found that, within two or three per cent,

$$\tilde{d}_p \simeq \left(\frac{\int_0^{100} d_p^2 dN}{100} \right)^{\frac{1}{2}} \simeq \left(\frac{\int_0^{100} d_p^3 dN}{100} \right)^{\frac{1}{3}} . \qquad (2)$$

Thus, the specific-area mean, area mean, and volume mean microscope size were not significantly different.

RESULTS

Table 1 gives the analyses of the coals used. They range from a high rank anthracite to a low rank bituminous coal.

Figure 1 shows the relation between $\tilde{d}_p$ and $\tilde{\mu}$ determined for sieve ranges of 35 x 50, 50 x 70, 70 x 100, 100 x 120, 120 x 140, 140 x 170, 170 x 200, 200 x 230, and 230 x 325 U.S. sieve numbers. For sieve sizes smaller than 80 microns (approximately 170 mesh), the ratio R appears to be constant at 1.68, with no significant difference between the coals tested. For sieve sizes larger than 80 microns, however, the curve bends over sharply, giving a second straight line section which does not pass through the origin. The variability of the results tends to obscure any differences between the coals. The best-fit curve above a sieve size of 80 microns has the equation

$$d_p = 40 + 1.18\mu \qquad (3)$$

or

$$\mu = \frac{d_p - 40}{1.18} . \qquad (3a)$$

The disadvantage of using $\tilde{\mu}$ to correlate with $\tilde{d}_p$ is that any error in measurement of the specific surface, S_o, of the sieve fraction considered will reflect as an error in $\tilde{\mu}$. This is in addition to the error involved in microscopic measurements, which gives rise to inaccuracies in the $\tilde{d}_p$ values. If the arithmetic mean sieve size of the fraction is used, the surface area error is avoided, but no allowance is made for the size distribution within the fraction. Figure 2 shows the relation between the volumetric-mean microscope diameter (which is close in value to $\tilde{d}_p$, see Equation (2)) and the arithmetic mean sieve size. The general form is the same as Figure 1, but there appears to be significant differences between coals for the portion of the curve above 80 microns.

Figure 3 gives values of R as a function of microscope size for the best fit curves of Figures 1 or 2. Values of sieve size calculated using these values of R and measured microscope diameters lie within ±10% of the arithmetic mean sieve sizes for all the results obtained.

DISCUSSION OF RESULTS

Previous results (9) indicated no change in the shape factor k with size for a given coal in the range 40 to 600 microns, yet R decreased significantly above sieve sizes of 80 microns. This result appears to be in conflict with Heywood's prediction (6). Further, the values of R obtained for the larger sizes were lower than those reported by other workers (3,7).

The material below 170 mesh size was viewed in ethyl cellulose suspension; but it would be expected that this would tend to reduce R rather than increase it, since the thinner sides of the particles might lie in the line of sight. It is clear that the curve of d_p against μ must pass through the origin. Therefore, even if the minus 170 mesh results were not available, it would be predicted that the curve must bend towards the origin. The shape of the curve is not caused by errors in area measurements, since Figure 2 is substantially the same even though surface area measurements play no part in its compilation.

It would appear that the method of size reduction of the particles has a considerable influence on the values of R, since the values obtained by Guruswamy and co-workers (7) were significantly higher than the results presented here. It would also appear that, for the narrow size ranges used, the use of a volume-mean microscope diameter and an arithmetic-mean sieve size is at least as satisfactory as using specific area mean diameters.

Although the coals used had specific-surface-area-shape factors, k, differing by as much as 30% (9), a significant correlation between the values of R and k was not clear. A possible explanation for this is as follows. If the ground coals have about the same length and breadth ratio, n, but varying thickness to breadth ratios, m, then it is easily shown that the values of k may vary widely with a comparatively small change in R. For instance, assuming a rectangular prism shape with $n = 2$ and m varying from 1/0.3 to 1/0.5 (10), k will vary by almost 30% while R will vary by less than 10%.

It is fortunate that the extrapolation of the curves of Figures 1 and 2 to the origin gives a constant value of R for the sub-sieve fraction, since this is the size range of most significance. The use of a value of R of 1.68 enables sub-sieve microscope distributions to be joined on to the sieve size distribution, in the present work, where the coals were ground in a Hardgrove machine.

CONCLUSIONS

For coals ground in the standard Hardgrove test mill, the ratio of microscope "projected area" diameter to sieve size, for material finer than 170 U.S. sieve size was found to be 1.68 for all the coals tested. Above this size, the ratio appears to decrease with increasing size to a limiting value of about 1.2. Although there may be a significant variation of the values of R (at the larger sizes) with the type of coal ground, this variation appears to be less than ca. ±10% for the four coals tested.

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TABLE 1

ANALYSES OF COAL USED

Coal	B-19447	B-17790	B-19426	St. Nicholas Anthracite
Constituent	As used,%	As used,%	As used,%	As used,%
Moisture	1.5	0.8	0.5	1.6
Ash	16.5	7.8	14.5	9.3
Carbon	65.3(83.5)*	78.8(87.6)*	75.2(90.6)*	84.2(95.5)*
Hydrogen	4.7(5.9)*	4.8(5.1)*	3.9(4.5)*	2.4(2.2)*
Nitrogen	1.1	1.5	1.5	0.85
Sulfur	4.5	1.6	1.8	0.5
Oxygen (by difference)	6.2	4.7	2.6	1.1
Volatile Matter (D.A.F.)	42.4	29.2	17.9	4.5
Shape Factor (k)	9.6	8.0	7.2	9.3
Hardgrove Grindability Index	52	93	99	30

* Parr's basis

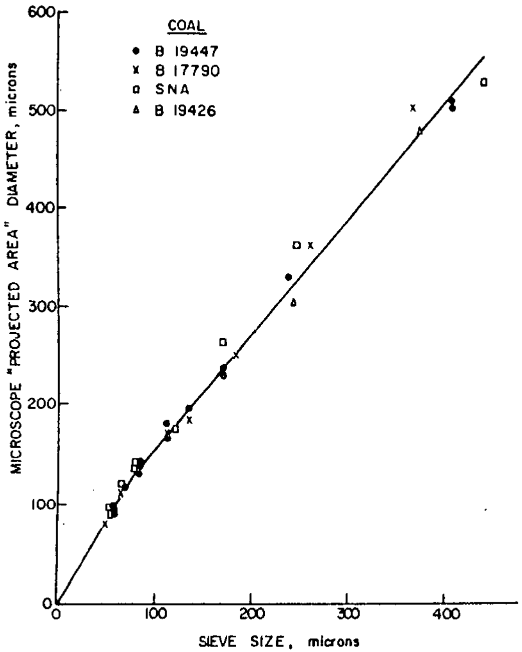

FIG. 1 – Relation between microscope diameter and sieve size for coals ground according to Standard Hardgrove Test.

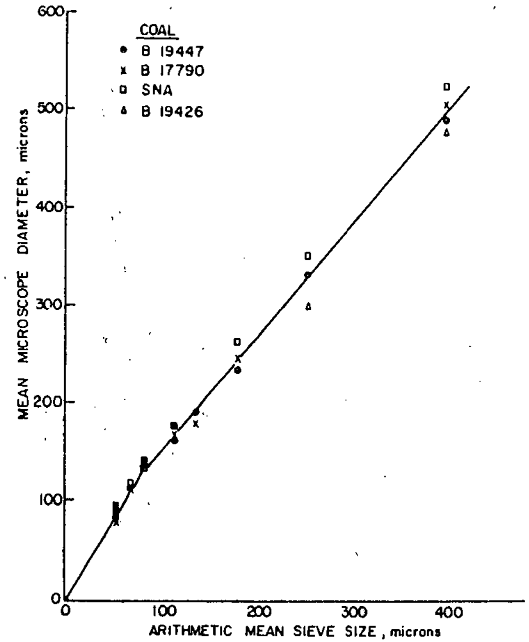

FIG. 2 – Relation of mean microscope size to arithmetic mean size.

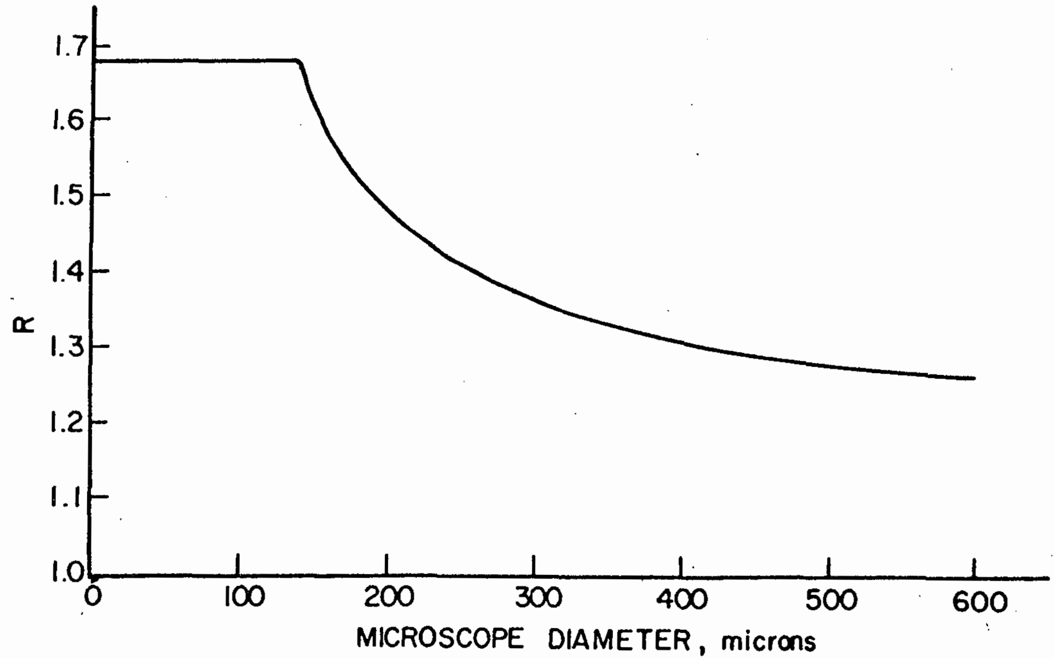

FIG. 3 — Change of microscope-size-to-sieve-size ratio (R) with microscope size for coals ground according to the Standard Hardgrove Test.