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The Production of Fresh Surface during the Grinding of Coal  
in a Standard Hardgrove Mill

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INTRODUCTION

For many years there has been considerable discussion as to whether the energy per unit volume required for size reduction of brittle materials is (a) proportional to the fresh area produced (Rittinger's Law) or (b) proportional to the reduction in volume of the particles (Kick's Law). Bickle (1) has given an excellent review of the available literature. For fine grinding, Rittinger's Law appears to be the law of most general application. Recent workers (2,3) consider such laws to be of limited utility in problems of mill design and operation. However, the relation of grinding energy to fresh surface area produced is of interest chemically as it offers a method of investigating surface energies of materials (4).

REVIEW OF PREVIOUS WORK

Although Rittinger's Law has been widely investigated for quartz, magnetite, and a variety of ores, there have not been many investigations of the law for coal. The primary reason for this is that coal contains an internal surface area (within micropores) that is large compared to the external area even for finely ground particles (5). Since most methods of area measurement measure, either completely or partially, this internal area, the increase in area upon grinding is obtained as the small difference between two large quantities. Consequently, the results are insufficiently accurate to be of much use.

Hardgrove (6) calculated proportional areas of ground coal from the sieve analysis. He assumed that shape factors and coal density remained constant throughout the size range and that shape factors were the same for different coals. The integration of the size distribution to a proportional area was carried out assuming that the minus 325 mesh size had a mean sieve size of 25 microns. Using these areas, he found the fresh surface produced to be proportional to the number of revolutions of the mill over a restricted range of revolutions. When large amounts of breakage had occurred, the increase in surface area on further grinding was less than that predicted by the increased number of revolutions; and Rittinger's Law did not hold. Hardgrove attributed this behavior to blanketing of the grinding by the fines produced. He defined the grindability of a coal in terms of the increase in surface area produced compared to that of the increase produced on grinding a standard coal the same number of revolutions, 60 revolutions being chosen as a standard condition. Hardgrove later found that there was an empirical relation between the grindability defined in this manner, and the per cent by weight of coal,  $p$ , passing a 200 mesh sieve, the relation being

$$\text{Hardgrove Index} = 13 + 3.465 p.$$

(1)

Romer (7) attempted to overcome some of the obvious objections to Hardgrove's work by measuring the surface areas of ground coals using an air permeability method. This method gave the hydrodynamic or geometric area of the particles. Romer found that the grindability indices calculated using the direct area measurements were considerably different from the Hardgrove indices, the surface areas being much higher than predicted for the products of coals of high Hardgrove indices. He then showed that Rittinger's Law applied when the load on the mill or the number of revolutions of the mill were varied. Thus, any non-applicability of Rittinger's Law in Hardgrove's original work was ascribed to inaccurate surface area measurements. Objections still remain to the surface areas obtained by Romer. The permeability method of area measurement is known to be inaccurate for a sample of mixed sizes in which the largest to finest size ratio is greater than 3 (8). Romer actually measured samples which consisted of coal of size range from one to 44 microns. Also, a certain amount of very fine material is lost during the grinding and sieving operations, and this surface area is not included in the measured area.

Bennet and Brown (2) argue that proofs of Rittinger's Law for coal are of little significance because the fresh surface area produced cannot be measured unequivocally.

#### EXPERIMENTAL PROCEDURES

Characteristics of Coals used in Tests - Four coals were used, ranging from an anthracite of 4.5% volatile matter to a low rank bituminous coal of 42.5% volatile matter and 6.2% oxygen content. The analyses were performed on the 16 x 30 mesh coal\* which was the starting material for all tests. The proximate and ultimate analyses were carried out according to A.S.T.M. standard procedures.

Preparation of Coal Samples - Since the interest was in the properties of the coal actually used and not in those of the bulk sample supplied, no studies were made on differences in character between the bulk sample and the final sample. The sample for use was prepared from minus 1/2-in. material by passing it through a jaw-crusher followed by a disc mill and sieving out the 16 x 30 mesh fraction on a Rotap sieving machine. The 16 x 30 mesh fraction was removed after each pass through the jaw crusher or disc mill. Microscopic examination showed the sample to be almost free from adhering fines or agglomerates. Before use, the coals were spread on trays in a thermostatically controlled (to  $\pm 0.5^\circ\text{C}$ .) laboratory and allowed to reach equilibrium with the atmosphere. Grinding and weighing were performed in this laboratory.

Grinding of Coal - The coals were ground in a standard Hardgrove test machine according to the A.S.T.M. standard method (9), both to measure the grindability and to provide sufficient fractions of material for surface area measurement. Two of the coals were also ground for varying revolutions of the machine, ranging from 3 revolutions, to 140 revolutions. In every test, 50 g. of coal were charged to the machine and the product sieved, as described below.

Sieving of the Ground Coal - It was considered essential that good performance of sieving be obtained; therefore, a standard procedure was carefully followed in each case. The material from the mill was carefully brushed out into the top sieve of a series of 6 sieves (16 mesh to 120 mesh). The sieves were shaken in

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\* All sieve numbers refer to U.S. standard mesh.

a Rotap sieving machine for 10 minutes, the material through 120 mesh removed, the sieve cleaned if necessary, and the sieves reshaken for five minutes. This was repeated for five minute intervals until the amount of minus 120 mesh material coming through was small. (A total sieving time of 25 minutes was always sufficient). The same procedure was then followed using the minus 120 mesh material in another series of 6 sieves (120 to 325 mesh). The sieves required cleaning more frequently and a maximum sieving time of 35 minutes was sometimes required. Cleaning was carried out by separating the sieves a small amount, inserting a brush and brushing the underside of the top screen. The collected sieve fractions were weighed to the nearest 0.01 g.

It was found that this series of multiple sievings gave weight losses outside of the tolerance given in the A.S.T.M. standard. Therefore, for standard Hardgrove Index determinations, the ground coal was sieved through a 200 mesh sieve only and the multiple sieving performed after the initial weighings. When this was done, the weight loss in the single sieving operation was within tolerance; and if the weight loss in the multiple sieving was assumed to be of material below 200 mesh, the Hardgrove Index was the same as that for the single sieving, within the tolerance allowable ( $\pm 2\%$ ).

When the coal was ground for a few revolutions only and the amount of fine material formed was small, the minus 120 mesh material was sieved through tared 3 inch diameter sieves. The coal sample plus sieve was then weighed directly to the nearest 5 milligrams

Surface-Area Shape Factors and Apparent Density of Coal Fractions - The shape factors and densities of ground coal fractions were determined as described in an earlier paper (10).

Size Distributions of Sub-Sieve Fractions of Ground Coal - To extend the cumulative weight versus sieve size to sub-sieve size particles, 0.5 g. of the minus 325 mesh fraction was sedimented into a fine and a coarse fraction and microscope sizing carried out on each fraction. The "sinks" obtained after repeated sedimentation with a 1/2 hour settling period were found to be free from any appreciable quantity of fine material. The "floats" were filtered, dried and weighed. Slides of each fraction were prepared and microscopic counts were performed on each fraction, as described in an earlier paper (11). The magnifications used were x 100 on the "sinks" and x 600 on the "floats". The sink material had microscope diameters mainly from 10 to 80 microns in size, and the float material ranged in size from less than 0.8 microns to about 20 microns. A cumulative weight against size distribution was calculated (see theory) for each fraction; and since the respective weight of each fraction was known, a combined distribution could be calculated. Microscope diameters were converted to sieve sizes using the correlation found previously (11).

#### THEORY

Calculation of Weight Versus Sieve Size Distributions for Sub-Sieve Coal Fraction from Microscopic Measurements - From microscopic count measurements, the cumulative per cent number of particles,  $N$ , below a given microscope size was determined as a smooth function of microscope size,  $d_p$ . By plotting  $N$  against  $d_p^3$ , the percentage weight  $p$  below a given size  $d_p$  was obtained graphically

since

$$\left(\frac{p}{d_p}\right) = 100 \frac{\int_0^{d_p} d_p^3 dN}{\int_0^{100} d_p^3 dN} \quad (2)$$

This assumes, of course, that the weight of a particle is proportional to the cube of its microscope diameter, but this assumption appears to be justified (12). The microscope size was then converted to sieve size by dividing by 1.68 (11).

Compilation of Accurate Sieve Size Versus Percentage Weight Undersize Curves -

When experimental results of weight versus size for varying revolutions were plotted, the results were not very consistent. This was due to the inherent variability of the grinding and sieving operations and inaccuracies in sieve sizes. The results were made more consistent by cross plotting the percentage weight below a given size against revolutions of grinding, drawing the best fit curve to the points, and taking values from the curve for a replot of weight versus size. This technique was found to give a very consistent family of curves, which could be extrapolated accurately to sub-sieve sizes.

Surface Area Change on Grinding - The surface area of ground coal was determined from the percentage weight versus size distribution, the values of shape factor  $k$ , and the density of the coal. If  $p$  is the percentage by weight below size  $\mu$ , then the experimental data on size distribution may be expressed graphically in the form,  $p = F(\mu)$ . As shown (10),  $dS = d_p k/\mu\rho$ . Therefore, the hydrodynamic area of coal between  $\mu_1$  and  $\mu_2$  is given by

$$S_{\mu_2-\mu_1} = \int_{\mu_1}^{\mu_2} \frac{k}{\mu\rho} dp \quad (3)$$

or

$$S_{\mu_2-\mu_1} = 2.3 \int_{\mu_1}^{\mu_2} \frac{kp}{\mu\rho} d(\log p) \quad (4)$$

From the experimental values,  $\log p$  can be plotted against  $\log \mu$ ; and  $p$ ,  $\mu$ , and  $\log p$  may then be obtained for any value of  $\mu$ . Since  $k$  and  $\rho$  are also known for this size,  $kp/\mu\rho$  may be plotted against  $\log p$  and  $S$  determined from the area under the curve.

Alternatively, if

$$\frac{d(\log p)}{d(\log \mu)} = n \quad (5)$$

then

$$S_{\mu_2-\mu_1} = 2.3 \int_{\mu_1}^{\mu_2} \frac{k p n}{\mu \rho} d(\log \mu) \quad (6)$$

This is somewhat more convenient as it allows a direct integration between any required sieve sizes. The values of  $n$  for any value of  $\log \mu$  are determined by taking the slope of the  $\log p / \log \mu$  distribution plot at that point. Below sieve sizes of 200 microns,  $n$  was found to be constant.

Over the part of the distribution for which  $n$  is a constant

$$p = B \mu^n \quad (7)$$

where  $n$  and  $B$  may be determined from the slopes and intercepts of the curve.

Now

$$S_{\mu_2-\mu_1} = \int_{\mu_1}^{\mu_2} \frac{k}{\mu \rho} \left( \frac{dp}{d\mu} \right) d\mu \quad \text{(sq. meters when } \mu \text{ is in microns)}$$

But from (7),  $dp/d\mu = B n \mu^{n-1}$ .

Therefore,

$$S_{\mu_2-\mu_1} = \int_{\mu_1}^{\mu_2} \frac{k B n}{\rho} \mu^{n-2} d\mu$$

When  $k$  and  $\rho$  are constant,

$$S_{\mu_2-\mu_1} = \frac{k B n}{\rho (1-n)} \left[ \frac{1}{\mu_1^{1-n}} - \frac{1}{\mu_2^{1-n}} \right] \quad (8)$$

Kick's Law Calculations - Kick's Law may be expressed in the form (13)

$$E = C \ln \left( \frac{\mu_1}{\mu_2} \right) \quad (9)$$

where  $E$  is the energy per unit weight required to reduce material of size  $\mu_1$  to size  $\mu_2$ , and  $C$  is a constant for a given material and process. If material of size  $\mu_1$  is broken to a distribution of sizes, the energy required to produce a weight  $dp$  of material of size  $\mu + d\mu$  is given by

$$dE = C \ln\left(\frac{\mu_1}{\mu}\right) dp \quad (10)$$

Therefore, the total energy is given by

$$E = C \int_{p=0}^{p=100} \ln\left(\frac{\mu_1}{\mu}\right) dp \quad (11)$$

It can readily be shown that when the energies required for various size reductions from the same starting material are to be compared, the precise value of  $\mu_1$  is not too important. Therefore, it was assumed that the 16 x 30 mesh starting material had a mean size  $\mu_1$  of 900 microns. Comparative values of E were obtained for various revolutions of grinding by plotting  $\log 900/\mu$  against p (using the appropriate size distribution of the product) and integrating graphically. The areas were not significantly different when lower size limits of 1 or 0.1 micron were assumed.

#### RESULTS

Table 1 gives the analyses of the coals tested. Figure 1 shows the size weight distributions of coal B-19447, after correcting the results as described previously. The points in Figure 1 are very consistent and the distribution curves can be drawn with considerable precision. If the straight line portions of the curves are extrapolated and the values of n and B (see Equation 7) determined, then the results are again very consistent, as can be seen from Figure 2.

Table 2 gives the cumulative weight versus microscope size (and corresponding sieve size) calculated using Equation (2), expressed both as a percentage of the minus 325 mesh sample tested and as actual weight of the minus 325 sample. The weight loss on grinding (60 revolutions) and sieving was 0.66 g. per 100 g., and it was assumed that this loss was in the very fine material. It was added, therefore, to the cumulative weight down to 1.5 microns.

Figure 3 shows a complete sieve size-weight distribution for the coal tested, using a factor of 1.68 to convert microscope size to sieve size (11). It can be seen that over the range 3 to 300 microns, the distribution is a straight line on the log-log plot. This type of distribution has been noted previously (14,15,16) and extended below sieve sizes by air elutriation. Rosin and Rammler (17) used air elutriation to extend results to sub-sieve sizes and concluded that the distribution obeyed the Rosin-Rammler law. However, for small sieve sizes, the Rosin-Rammler distribution becomes the simple power distribution found in Figure 3. (In fact, a size distribution of broken coal (18) over a range of 0.004 to 5 inches which fits a Rosin-Rammler plot, also fits a log p-log  $\mu$  plot over most of the same range). The departure of the curve from the straight line below three microns is almost certainly due to the assumption that all of the weight loss on grinding is less than 1.5 microns. The break in the curve suggests that the weight loss is in sizes of about three microns (sieve size) and less.

Figure 4 shows the surface areas of the ground coal fractions for coal B-19447 calculated using Equations (4), (5), and (8). The surface areas were

calculated assuming that the straight line part of the log  $p$  versus log  $\mu$  distribution (Figure 3) could be extrapolated to a lower limit of 1, 0.1, or 0.01 micron. It was also assumed that the shape factor (a constant in the range from 40 to 600 microns (10)) was constant down to the lower size limit. It is clear that the lower size limit which is chosen considerably affects the absolute value of the surface area. The curve is a straight line when a lower size limit of about 1 micron is used. For the lower size limits of 0.1 and 0.01 micron, the area increases more with revolutions of grinding than Rittinger's Law would predict. Another feature of interest is that the extrapolation of the curves to zero revolutions gives an initial surface area of unground material of 1.2 m.<sup>2</sup>/100 g. whereas the actual unground surface area is 0.8 m.<sup>2</sup>/100 g. It appears that an initial small amount of grinding produces 0.4 m.<sup>2</sup> of surface area per 100 g. in addition to the 0.117 m.<sup>2</sup>/100 g./revolution produced for the remainder of the grinding process.

Figure 5 shows the surface area change with grinding for coal B-17790, where a lower limit of 1 micron has been used. After an increase to about 16 m.<sup>2</sup>/100 g., it appears that the increase in surface area is no longer proportional to the revolutions of grinding. Extrapolation of the straight line portion of the curve to zero revolutions again indicates an initial area of 1.2 m.<sup>2</sup>/100 g. instead of the expected value of 0.8 m.<sup>2</sup>/100 g.

Table 3 gives the surface areas from 1 micron to 1190 microns for the four coals ground according to the standard Hardgrove test and also gives the increase in surface areas on grinding. For coal B-19426 and the St. Nicholas anthracite, the results are based on measurements at 60 revolutions only; the cross plotting technique was not used as the data were not available.

Figure 6 gives the increase in  $E$  (Equation 11) with revolutions of grinding for coal B-19447. It can be seen that  $E$  is not proportional to revolutions of grinding over wide ranges of grinding. Therefore, Kick's Law does not appear to hold for grinding in accordance with the standard Hardgrove test.

#### DISCUSSION OF RESULTS

Rittinger's Law cannot in general be true, which can be seen if an extreme case is considered as follows. Let a grinding machine be grinding particles which have such strength that the grinding forces imparted by the machine do not exceed this strength. Work will be done without the production of fresh surface, but the material does not have an infinite strength, since a heavier machine would produce breakage. If no change of state of the material occurs, then the energy input is dissipated in a variety of ways. There will be the loss of energy as frictional slip over the surface of the ground material. Also, particularly for ball mills, forces will be transmitted through the material to deliver blows on the mill structure; and energy will be lost as heat of impact and impact waves. In both of these cases, the energy finally appears mainly as heat. In addition, when the ground material fractures, energy will be used to break force bonds across the fresh surface produced and to form internal cracks and flaws. Energy will also be liberated as heat of fracture.

The process of fracture may be loosely described in the following manner. When a particle of coal is crushed it must be raised to a strained state before it fractures. (This, in effect, is an activation energy for crushing). Energy is imparted by the grinding forces which are applied over distances corresponding to the deformation of the particle. The coal then breaks at a flaw

or series of flaws in the material, deformation is removed, and fracture waves propagate through the coal producing fresh surface (19). The excess energy of the fracture waves and the energy released on the relaxation of deformation appear eventually as heat of fracture.

From the above discussion it would appear unlikely that there would be a simple relation between the fresh surface produced and the total energy input to the grinding process; there are so many different ways in which the energy can be distributed. It seems plausible, however, to assume that under certain conditions the energy lost as frictional slip and impact is a fixed fraction of the total input. Fresh internal area does not seem to be produced; except, perhaps, in direct proportion to the external area (20). A fracture wave will propagate until it reaches a free surface and it will not end within the material. (Gross and Zimmerly (21) found that for quartz, internal area was broken out on grinding and impact crushing, rather than increased). Thus, if the energy used to produce fresh surface is a fixed proportion of the strain energy, which in turn is a fixed fraction of the total energy input, Rittinger's Law would hold. To investigate the surface to strain energy relation further, a distinction can be made between the "strength" of a coal and its "hardness". The strength is here arbitrarily defined as the strain energy required before a particle fractures (which is a function of the type of forces imparting this energy to the particle). The hardness is arbitrarily defined as the strength of surface bonds in the material. Clearly two particles may have the same composition and hence the same hardness but may have widely different strengths, if one is highly flawed and the other not.

Consider two such particles of similar "hardness" but different strengths. The stronger one will require the addition of more energy to fracture it, but it seems possible that on fracture it will break into many smaller pieces. On the other hand, the weaker particle will break more readily with a lower energy content but will break into fewer pieces with correspondingly lower fresh surface. Similarly, a large impulsive force of low energy application might cause breakage with small area production; whereas a smaller force applied for a much longer time and deformation would produce a larger surface area on eventual shatter. Thus it is possible that particles of entirely different strengths, and hence probabilities of breakage, have breakage functions which automatically compensate, so that the fraction of the strain energy which is used to produce fresh surface is constant. Bickle (22) states that this has been considered theoretically, but gives no reference to such studies. Such a concept would go part way toward explaining the validity of Rittinger's Law with progressive grinding, although the strength of coal particles is known to vary with size (23,24) and degree of grinding. This concept implies that grindability indices based on surface-area increase measure a parameter proportional to hardness rather than a combined effect of hardness and strength. It is interesting to see that there is a pronounced correlation between grindability indices and the Vickers Microhardness test (25).

As particles become smaller and stronger (in comparison to their size) on grinding due to the breaking out of flaws, they may eventually reach a stage where the crushing forces of the machine are insufficient to cause much breakage. Grinding experience indicates that it is extremely difficult to reduce anthracite below 0.1 to 1 micron in size in conventional grinding apparatus. It may be postulated that somewhere near this size range the major flaw structure of the coal has been completely broken out and that grinding is more difficult by an order of magnitude or more. Van Krevelen (26) states that Boddy found coal particles to be initially crushed to 1 micron in size. As the surface area of this material is of the same order as the macropore area of unground coal, Van Krevelen suggests that breakage to 1 micron is favored by the macropore system. For smaller particles, the coal tends to plastically deform rather than fracture;

this implies much greater strength and a low grindability. An alternative hypothesis is that agglomerates of fine particles tend to trap air and on grinding behave somewhat like miniature balloons. The crushing force is applied to the "balloon" and the energy is expended in compressing the contained gas. For grinding in liquid media, the fluid is incompressible and the forces are imparted to the coal particles; it is well known that liquid grinding can be used to produce very fine sizes.

In spite of the theoretical objections to Rittinger's Law, there is considerable evidence that under a restricted range of conditions the Law is closely obeyed. The correlation of the increase in surface area with grinding obtained in this work is not conclusive, since the lower size limit chosen for the integration to obtain surface area is rather arbitrary. Microscopic studies of the fine fractions of ground coal indicated that material below 1 micron in size was not present in large quantities, although there still remains the question of the fineness of the material making up the weight loss on grinding and sieving. From electron micrographs of ground coal, Preston and Cuckow (27) conclude that coals ground in the normal manner had few particles of less than about 1 micron in size. By taking a lower limit of 1 micron, it is not assumed that material less than 1 micron is absent but rather that 1 micron represents an effective lower limit for the straight line  $\log p/\log \mu$  distribution extrapolated from the sieving results. The strict linearity, over a fairly wide range of grinding, of the results plotted in Figure 4 for the 1 micron lower limit would hardly occur by coincidence; and it must be concluded that the evidence for the accuracy of Rittinger's Law is quite strong.

Figure 7 shows the Hardgrove Grindability Indices of a number of British coals (15) and the four coals tested in this work, as a function of a rank index (10). It is clear that the grindability characteristics of a coal are closely allied to its rank. Although the British coals (because they are of one geological era) might be expected to form a fairly consistent pattern, the coals used in our experiments fit the mean line with as good an accuracy as the British coals. Deviations from the mean line are quite considerable in some instances, more than would be expected by experimental error of determination of C, H, or Hardgrove Index. This may be due to several causes:

- a) The mineral matter of a coal might considerably influence its grindability.
- b) Grindability, as measured by the Hardgrove Index, might not be an accurate representation of the grinding strength of the coal.
- c) Differences in the amounts of macerals present in the coal might cause considerable change in strength.
- d) The grindability might be influenced by factors which do not depend closely on rank, for example flaw structure.

At the moment, it is only possible to discuss cause (b) with knowledge obtained from our own results. Figure 8 shows the per cent by weight less than 200 mesh plotted against surface area for varying revolutions for coals B-19447 and B-17790. Clearly the increase in surface area is not related to  $P_{200}$  in the form of Equation (1). The surface area is not linearly proportional to  $p$  for coal B-17790, although a straight line could be drawn with a fair degree of accuracy.

Figure 9 shows  $P_{200}$  plotted against increase in surface area for the four coals ground for 60 standard revolutions, and it also shows the Hardgrove Index as a function of surface area increase. It can be seen that the Hardgrove Index is not proportional to the increase in surface area. (Surface areas used are

those calculated on the assumptions that the shape factor is constant over the range 1 to 1190 microns and that 1 micron is an effective lower limit; it has only been shown that the shape factor is constant over the range of 40 to 600 microns (10)).

From Figure 9 it can be seen that the increase in surface area is proportional to  $p_{200}$  only to a degree of accuracy of about  $\pm 10\%$ . This is of the same order as the deviations of Hardgrove Index values from the best fit curve in Figure 7, and it is possible that part of the deviations are caused by the Hardgrove Index (which depends on  $p_{200}$ ) not being an accurate representation of the increase in surface area. This is particularly likely to be true where a coal fractures to give products with an abnormal shape factor, for in this case two coals might have very similar size distributions on grinding but would have considerably different surface areas.

Callcott (28) argues that the increase in surface on grinding is of little significance in practical grinding studies. He analyzes the problem of grinding in the following manner: Given different sized feeds into a grinding machine, what will be the size distributions of the products? Or if different machines operate with different size feeds, how much of the difference in products is due to the different feed sizes and how much is due to differences caused by the machines? Callcott suggests using  $p$  as an index of grindability in preference to the Hardgrove Index (this was also suggested by Frisch and Holder (29)). He does not believe that the work on surface area increase during grinding justified the use of any index except a simple index of breakage defined by  $p$ . The significance of the grindability index  $p$  may be stated in these terms: If a certain coal produces 10 per cent of material below 200 mesh in the standard Hardgrove test and another coal produces 20 per cent, then it is likely that on grinding in an industrial mill, the first coal will have approximately half the throughput of minus 200 mesh material obtained with the second.

Thus, it would appear that  $p_{200}$  is a better index of grindability than the Hardgrove Index, both for the reasons given by Callcott and because it is a better index of surface area increase. The Hardgrove Index may be used instead of  $p_{200}$ , if it is borne in mind that a Hardgrove Index of 13 represents zero production of fresh surface. For scientific work it is recommended that the index used should be the increase in surface area per revolution of grinding (over the range in which linearity is obtained).

#### CONCLUSIONS

The log  $p$  versus log  $\mu$  straight line portions of the distributions found for coals ground according to the Hardgrove test can be extrapolated to at least 3 microns sieve size. The weight loss on grinding appears to be mainly material of less than 3 microns sieve size.

For the two coals tested at varying revolutions of grinding, a negligible amount of grinding produced about  $0.4 \text{ m.}^2/100 \text{ g.}$  of fresh geometric surface; but after this initial abnormal increase, the increase in surface area was proportional to the revolutions of grinding up to the condition of at least 20% of the material through a 200 mesh sieve. This was true when a lower limit of size of about 1 micron was used to calculate the surface area. Kick's Law did not apply. The percentage of material passing a 200 mesh sieve is very approximately proportional to the increase in surface area on grinding. More precise values of surface area increase per revolution of grinding will be obtained in future work, and these values compared to the rank of the coals used.

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TABLE I  
ANALYSES OF COAL USED

Coal	B-19447	B-17790	B-19426	St. Nicholas Anthracite
Constituent	As used, %	As used, %	As used, %	As used, %
Moisture	1.5	0.8	0.5	1.6
Ash	16.5	7.8	14.5	9.3
Carbon	65.3(83.5)*	78.8(87.6)*	75.2(90.6)*	84.2(95.5)*
Hydrogen	4.7(5.9)*	4.8(5.1)*	3.9(4.5)*	2.4(2.2)*
Nitrogen	1.1	1.5	1.5	0.85
Sulfur	4.5	1.6	1.8	0.5
Oxygen (by difference)	6.2	4.7	2.6	1.1
Volatile Matter (D.A.F.)	42.4	29.2	17.9	4.5
Shape Factor (k)	9.6	8.0	7.2	9.3
Hardgrove Grindability Index	52	93	99	30

\* Parr's basis

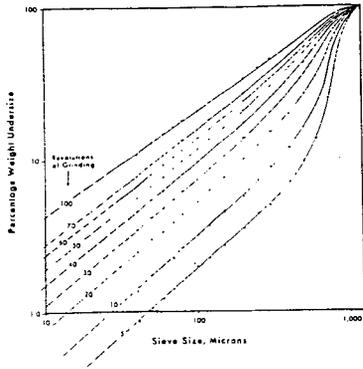
TABLE 3  
HYDRODYNAMIC SURFACE AREAS OF COALS GROUND ACCORDING TO THE STANDARD HARDGROVE TEST

Coal	Hardgrove Grindability Index	Surface Area ground coal m. <sup>2</sup> /100 g.	Increase in Surface Area m. <sup>2</sup> /100 g.	Rank-Index of coal % C - 8.5 % H
B-19447	52	7.6	6.8	33.5
B-17790	93	15.9	15.1	44.2
B-19426	99	14.5	13.7	52.4
St. Nicholas anthracite	30	3.4	2.6	77.0

TABLE 2

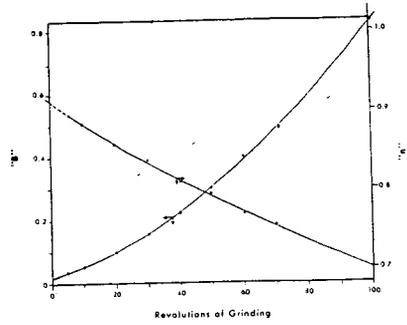
SIZE DISTRIBUTION BELOW 325 MESH OF COAL B-19447 GROUND  
 ACCORDING TO STANDARD HARDGROVE TEST

Microscope diameter, microns	Equivalent sieve size, microns	Cumulative % by weight of -325 fraction tested	Cumulative weight expressed as a % of total coal ground	Plus 0.66% weight loss of fine material on grinding
0	0	0	0	
0.7	0.4	0.0046	0.00035	
0.9	0.5	0.014	0.00104	
1.3	0.8	0.049	0.00356	
1.8	1.0	0.15	0.0108	
2.6	1.6	0.56	0.0408	(0.701)
3.5	2.1	1.24	0.090	(0.750)
4.4	2.6	2.18	0.16	(0.819)
5.8	3.5	3.67	0.27	0.927
7.3	4.3	6.00	0.44	1.10
8.8	5.2	8.51	0.62	1.28
10.2	6.1	9.90	0.72	1.38
13.1	7.8	14.7	1.07	1.73
21.0	12.5	29.2	2.12	2.78
26.2	15.6	36.1	2.62	3.28
35.0	20.8	44.9	3.26	3.92
43.6	26.0	56.0	4.07	4.73
52.5	31.0	70.6	5.14	5.80
61.0	36.4	83.8	6.10	6.76
70.0	41.6	93.7	6.82	7.48
78.6	47.0	100.0	7.28	7.94



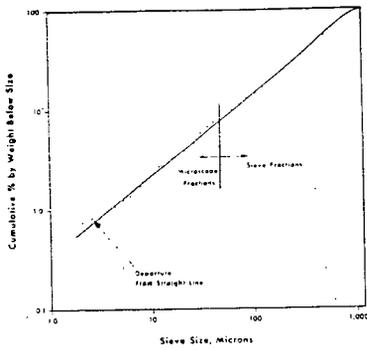
CORRECTED PERCENTAGE WEIGHT UNDERSIZE VERSUS SIEVE SIZE DISTRIBUTIONS FOR COAL B-19447

Figure 1



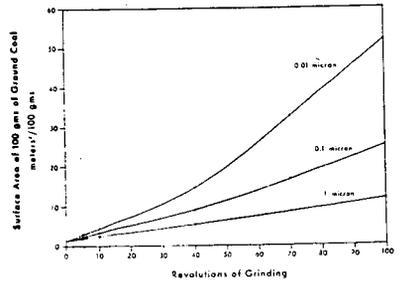
VALUES OF "n" AND "B" IN  $D = B\mu^n$  FOR COAL B-19447 GROUND FOR VARYING REVOLUTIONS

Figure 2



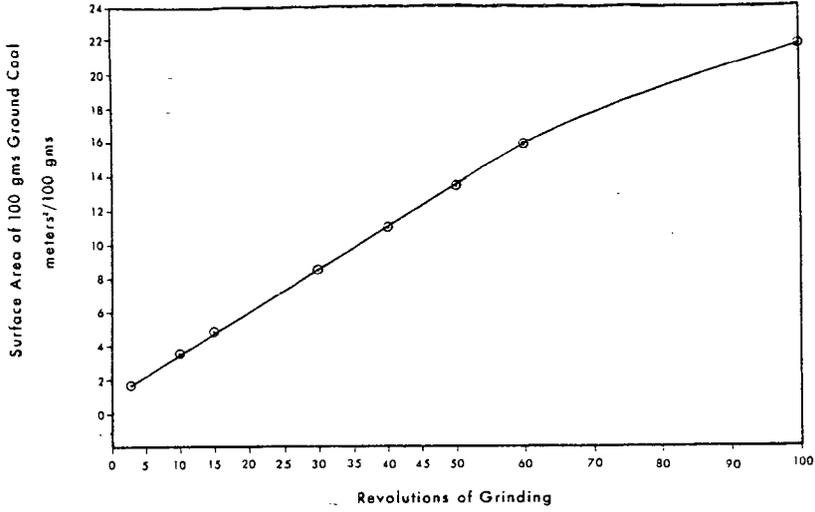
EXTENDED PERCENT WEIGHT UNDERSIZE VERSUS SIEVE SIZE DISTRIBUTION FOR COAL B-19447 GROUND FOR 60 REVOLUTIONS

Figure 3



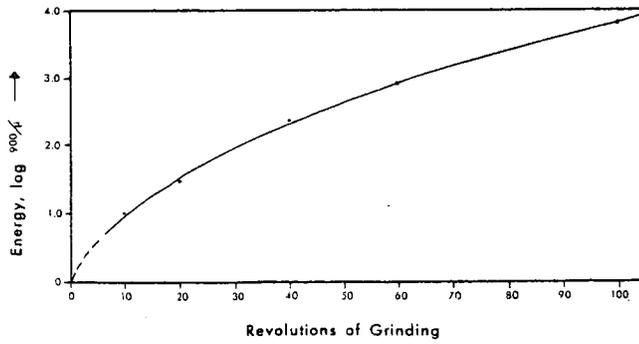
INCREASE OF HYDRODYNAMIC SURFACE AREA OF GROUND COAL WITH REVOLUTIONS OF GRINDING FOR COAL B-19447, ASSUMING 1, 0.1 AND 0.01 MICRONS AS THE SMALLEST SIZE PRESENT

Figure 4



INCREASE OF HYDRODYNAMIC SURFACE AREA OF GROUND COAL WITH REVOLUTIONS OF GRINDING FOR COAL B-17790, ASSUMING 1 MICRON AS LOWER LIMIT

Figure 5



RELATION OF ENERGY FOR GRINDING PREDICTED BY KICK'S LAW TO REVOLUTIONS OF GRINDING (COAL B-19447)

Figure 6

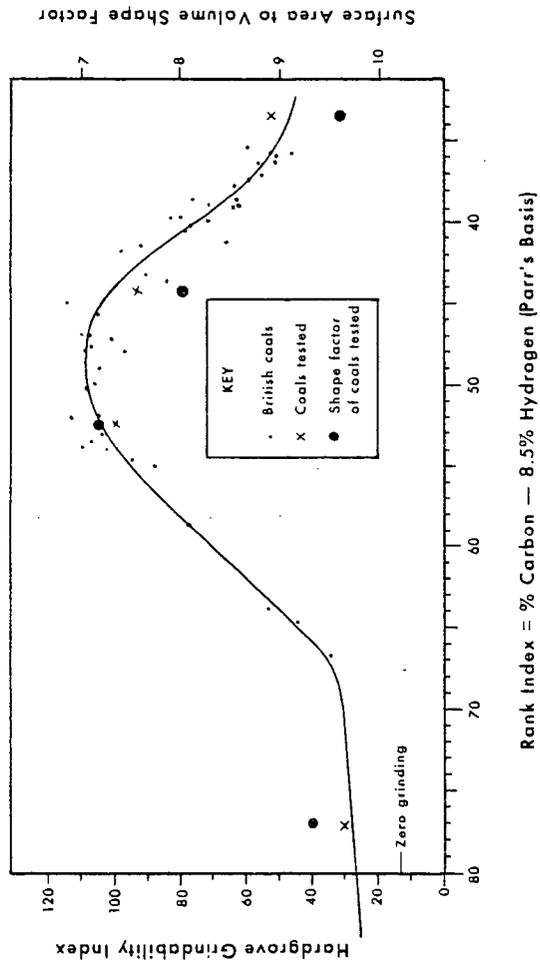
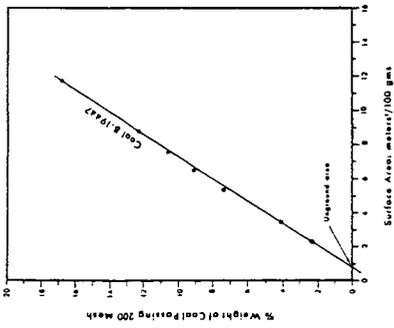
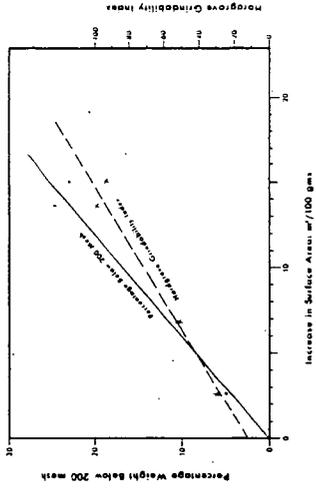


Fig. 7. GRINDABILITY INDEX AND SHAPE FACTOR AS A FUNCTION OF COAL RANK.



RELATION OF SURFACE AREA TO PERCENT WEIGHT THROUGH 200 MESH FOR VARYING REVOLUTIONS OF GRINDING IN THE HARDGROVE MACHINE

Figure 8



INCREASE IN SURFACE AREA ON GRINDING ACCORDING TO STANDARD HARDGROVE TEST AS A FUNCTION OF % WEIGHT THROUGH 200 MESH

Figure 9

