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POTENTIAL DISTRIBUTION AND INTERNAL RESISTANCE IN ELECTROCHEMICAL MATRIX CELLS WITH DISCONTINUOUS CONTACTS

By

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I. INTRODUCTION

Recent advances in the field of electrochemical fuel cells have brought about renewed interest in cells employing either solid electrolytes or solid matrices holding electrolytes. Thus interest stems from a tendency to avoid the difficulties resulting from the use of fluid electrolytes, such as "flooding" of the electrode and other problems in the maintenance of a 3-phase equilibrium at a gaseous electrode. These difficulties are readily avoided when a solid-matrix electrolyte is employed, such as found in ion-exchange membrane cells or the Ketelaar-type high temperature fuel cells. On the other hand, the use of such electrolytes involves two other possible difficulties.

One is the practical problem of assurance that the electrode material makes a uniform contact with the electrolyte matrix throughout the area; and the other, the more fundamental problem of an increased effective cell resistance, resulting from the necessity of employing discontinuous contact electrodes to assure satisfactory access of the gaseous reactant. It is the latter problem that this paper is concerned with.

In practice, the contact electrodes are in the form of either a screen pressed against the electrolyte matrix or membrane by means of backup plates or in the form of perforated metal sheets. The interesting question is then, how does the resistance of such a cell compare to that of a cell of identical dimensions which would employ solid continuous electrodes? A mathematical treatment of this problem requires idealization of geometry. Thus, for the case of screens, one can assume that the contact points are in the shape of either squares or circles located on square centers; for the case of perforated plates, the contact area may be that left after the punching of square or round holes on square centers in a continuous sheet.

The problem of square contacts was first considered by Gorin and Recht (1). Their solution, basically correct, was found, however, to be slowly convergent (2), necessitating at least a 100 term computation for the summation terms. For very thin electrolyte layers and screens with a low ratio of contact area to total area, it was found, for instance, that computation of only 10 terms would lead to an error, in the resistance value, of the order of 100%.

This paper restates the solution for the case of square contacts and solves three other cases, namely circular contacts and contact electrodes with square holes and round holes on square centers. Two important aspects must be borne in mind. One is that the solution inquires only about the primary potential (or current) distribution. It does not deal with the more complex problem resulting from polarization contributions (i.e., with the secondary distribution problem). The other point is of a practical nature arising from the fact that the contact geometry is idealized for the sake of the mathematical models, amenable to an exact solution of the Laplace equation. In practice, however, metal powder or other catalysts are employed together with the screens or contact plates so that the actual cell resistance may be different. Still, however, information obtained from such computations is desirable for a first order estimate of the resistivity of the cells with discontinuous contact electrodes.

In order to bring the solutions of the four cases into the realm of practical interest, numerical computations of the exact solutions were performed, employing ranges of geometric parameters corresponding to those of practically available screens and plates.

TABLE I

RESULTS OF RESISTANCE RATIO CALCULATIONS

Electrolyte Thickness h (inches)	Dimension C (inches)	% Area Contact	RESISTANCE RATIO R_{eff}/R Square Contacts	Circular Contacts	Square Holes	Circular Holes
0.040"	0.004"	1	21.0170	16.7620		
		2	10.4728	9.4411		
		5	4.3832	4.3512	1.3549	
		10	2.4672	2.5215	1.2747	
		15	1.8644	1.9014	1.2302	
		25	1.4132	1.4222	1.1759	1.2215
		40	1.1836	1.1776	1.1276	1.1203
		50	1.1146	1.1052	1.1047	1.0861
		60				1.0624
		75	1.0332		1.0603	1.0372
0.040"	0.008"	1	40.8890	32.5241		
		2	19.9456	17.8824		
		5	7.7666	7.7024	1.7098	
		10	3.9345	4.0431	1.5494	
		15	3.2886	2.8029	1.4604	
		25	1.8264	1.8440	1.3519	1.4431
		40	1.3672	1.3552	1.2373	1.2406
		50	1.2293	1.2103	1.2094	1.1721
		60				1.1248
		75	1.0821		1.1207	1.0744
0.040"	0.010"	1	50.8612	40.4051		
		2	24.6819	22.1031		
		5	9.4583	9.3781	1.8873	
		10	4.6682	4.7848	1.6868	
		15	3.1611	3.2536	1.5755	
		25	2.0330	2.0555	1.4399	1.5539
		40	1.4590	1.4440	1.3191	1.3008
		50	1.2866	1.2629	1.2618	1.2152
		60				1.1560
		75	1.0831		1.1508	1.0929
0.040"	0.012"	1	60.8334	48.2862		
		2	29.4187	26.3237		
		5	11.1500	11.0537	2.0648	
		10	5.4018	5.5647	1.8241	
		15	3.5932	3.7043	1.6906	
		25	2.2396	2.2666	1.5278	1.6646
		40	1.5508	1.5328	1.3829	1.3610
		50	1.3439	1.3155	1.3142	1.2582
		60				1.1872
		75	1.0997		1.1810	1.1115
0.040"	0.016"	1	80.7779	64.0482		
		2	38.8915	34.7649		
		5	14.5332	14.4049	2.4197	
		10	6.8691	7.0862	2.0988	
		15	4.4577	4.6057	1.9214	
		25	2.6528	2.6888	1.7038	1.8862
		40	1.7344	1.7105	1.5106	1.4813
		50	1.4586	1.4207	1.4189	1.3443
		60				1.2496
		75	1.1330		1.2413	1.1487

II. MATHEMATICAL FORMULATION

(1) General

The effective internal resistance of the flat galvanic cell, illustrated in cross-section in Figure 1, may be computed from the potential distribution obtained from the solution of the Laplace equation:

$$\nabla^2 V = 0 \tag{1}$$

relating the potential V to the position in the electrolyte under steady state conditions. From the general solution for the potential, the cell resistance can be obtained by calculating the potential difference between the electrode-electrolyte interfaces (at positions $\frac{h}{2}$ and $-\frac{h}{2}$ in Figure 1). The effect of the cell and electrode geometry is best expressed as the ratio of the effective resistance (Reff) to the resistance (R) of a cell consisting of two solid continuous plane electrodes.

When A_c (see nomenclature) represents the conducting portion of the area element of size 2d x 2d, then the boundary condition at the contacts with the electrolyte is (see Figures 1 and 2):

$$\left.\frac{\partial V}{\partial z}\right]_{z = \pm \frac{h}{2}} = \frac{i\,\rho}{A_c/4d^2} = K \tag{2}$$

over the contact area Ac (with periodicity in the x and y directions) and

$$\left.\frac{\partial V}{\partial z}\right]_{z = \pm \frac{h}{2}} = 0$$

elsewhere on the electrode plane.

It should be noted that (see nomenclature and Figure 2) for screen electrodes (square and circular contacts) $A_c = A$; for perforated plates $A_c = 4d^2 - A$.

Due to the periodicity in the x and y directions, the general form of the potential, V, is assumed to be:

$$V = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_{nm}(z) \cos \frac{n\pi x}{d} \cos \frac{m\pi y}{d} + C_0 + C_1 \tag{3}$$

where n, m = 0, 1, 2, 3, ... in the x and y directions respectively. Substitution into equation (1) gives:

$$\nabla^2 V = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[\frac{\partial^2 a_{nm}}{\partial z^2} - \frac{(n^2+m^2)\pi^2}{d^2} a_{nm} \right] \cdot \tag{4}$$

$$\cdot \cos\left(\frac{n\pi x}{d}\right) \cos\left(\frac{m\pi y}{d}\right) = 0$$

The above indicates that the Fourier coefficient a_{nm} is of the form:

$$a_{nm} = b_{nm} \sinh \left(\frac{\pi z \sqrt{n^2+m^2}}{d}\right) \tag{5}$$

the sinh (rather than cosh) being chosen because of the necessity for an even function to express the relationship. Substituting (5) into (3) and taking the partial derivative with respect to z one obtains:

$$\left.\frac{\partial V}{\partial z}\right]_{z = \pm \frac{h}{2}} = C_1 + \frac{\pi}{d} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} b_{nm} \sqrt{n^2+m^2} \cos \frac{n\pi x}{d} \tag{6}$$

$$\cdot \cos \frac{m\pi y}{d} \cosh \frac{\pi h \sqrt{n^2+m^2}}{2d}$$

Electrolyte Thickness h (inches)	Dimension C (inches)	% Area Contact	RESISTANCE RATIO R_{eff}/R			
			Square Contacts	Circular Contacts	Square Holes	Circular Holes
0.080"	0.016"	1	40.8890	32.5241		
		2	19.9456	17.8824		
		5	7.7666	7.7024		
		10	3.9345	4.0431		
		15	3.2886	2.8029		
		25	1.8264	1.8440		1.4431
		40	1.3672	1.3552		1.2406
		50	1.2293	1.2103		1.1721
		60				1.1248
		75	1.0821			1.0744
0.080"	0.020"	1	50.8612	40.4051		
		2	24.6819	22.1031		
		5	9.4583	9.3781		
		10	4.6682	4.7848		
		15	2.3550	3.2536		
		25	2.0330	2.0555		1.5539
		40	1.4590	1.4440		1.3008
		50	1.2866	1.2629		1.2152
		60				1.1560
		75	1.0831			1.0929
0.120"	0.004"	1	7.5817	6.2013		
		2	4.1260	3.7856		
		5	2.1164	2.1059	1.1171	
		10	1.4842	1.5021	1.0907	
		15	1.2852	1.2975	1.0760	
		25	1.1363	1.1393	1.0581	1.0731
		40	1.0606	1.0586	1.0421	1.0397
		50	1.0378	1.0347	1.0346	1.0284
		60				1.0206
		75	1.0110		1.0199	1.0123
0.120"	0.008"	1	14.1633	11.4028		
		2	7.2519	6.5711		
		5	3.2329	3.2118	1.2343	
		10	1.9683	2.0004	1.1813	
		15	1.5705	1.5949	1.1519	
		25	1.2727	1.2786	1.1161	1.1462
		40	1.1212	1.1172	1.0842	1.0794
		50	1.0756	1.0694	1.0691	1.0568
		60				1.0412
		75	1.0219		1.0398	1.0245
0.120"	0.010"	1	17.5539	14.0824		
		2	8.8624	8.0062		
		5	3.5928	3.7815	1.2946	
		10	2.2178	2.2629	1.2280	
		15	1.7174	1.7482	1.1911	
		25	1.3429	1.3504	1.1460	1.1839
		40	1.5238	1.1553	1.1059	1.0999
		50	1.0951	1.0873	1.0869	1.0714
		60				1.0518
		75	1.0276		1.0501	1.0309

(4) Solution for Circular Contacts

For circular contacts of radius c on 2d centers (see Figure 2C) the expression for C_1 is:

$$C_1 = \frac{\pi}{4} K r^2 \tag{14}$$

and the expression for R_{eff}/R is:

$$\frac{R_{eff}}{R} = 1 + \frac{16d}{\pi^2 rh} \sum_{n=1}^{\infty} \frac{J_1(n\pi r)}{n^2} \tanh \frac{n\pi h}{2d} + \frac{16d}{\pi^2 rh} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{J_1(\sqrt{n^2+m^2}\,\pi r)}{n^2+m^2} \tanh \frac{\sqrt{n^2+m^2}\,\pi h}{2d} \tag{15}$$

Where J_1 is the Bessel function of the first kind of order 1.

(5) Solution for Plane Electrodes with Circular Holes

For this case (see Figure 2D), the expression for K is:

$$K = \frac{\rho}{(1 - \frac{\pi r^2}{4})} \tag{16}$$

since $A_c = 4d^2 - A$
and the expression for R_{eff}/R is:

$$R_{eff}/R = 1 - \frac{4c}{\pi h(1 - \frac{\pi r^2}{4})} \sum_{n=1}^{\infty} (-1)^n \frac{J_1(n\pi r)}{n^2} \cdot \tanh \frac{n\pi h}{2d} - \frac{4c}{\pi h(1 - \frac{\pi r^2}{4})} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^{m+n} \cdot \frac{J_1(\sqrt{n^2+m^2}\,\pi r)}{m^2+n^2} \tanh \left(\frac{\sqrt{n^2+m^2}\,\pi h}{2d} \right) \tag{17}$$

III. RESULTS OF COMPUTATIONS

The solutions of the single and double summations in equations (11), (13), (15) and (17), for the four electrode configurations outlined in the preceding section, all exhibit the same convergence characteristics. The value of the sums (both single and double) plotted against the number of terms computed, oscillate about the true solution with decreasing amplitude, and a periodicity which are a direct function of the argument r. Low values of r produce the maximum amplitude and minimum period. High values result in minimum amplitudes and maximum periods. A maximum of 100 terms (100^2 for the double summation) was required to establish the convergence characteristics sufficiently to determine the true solution for the case of very small r - values. Most of the other cases converged with a much lower number of terms. An example of this convergence characteristic is shown in Figure 3.

A computer program for the IBM 7090 was written for the solution of the summation terms involved in each of the four different electrode configurations. For this study, the convergence characteristics were then determined by inspection. The added sophistication of programmed convergence inspection was considered but not deemed worth while for the expected volume of production on the program.

An interesting question is how the distribution of contact positions (expressed by dimension c) per unit cross-sections area of cell would affect the cell resistance for a given fractional contact area. To investigate this point, the resistance ratio was also computed as a function of c for several values of fixed fractional contact areas.

Electrolyte Thickness h (inches)	Dimension C (inches)	% Area Contact	RESISTANCE RATIO R_{eff}/R Square Contacts	Circular Contacts	Square Holes	Circular Holes
0.120"	0.012"	1	21.0171	16.7620		
		2	10.4728	9.4411		
		5	4.3832	4.3512		
		10	2.4672	2.5215		
		15	1.8644	1.9014		
		25	1.4132	1.4222		1.2215
		40	1.1836	1.1776		1.1203
		50	1.1146	1.0425		1.0861
		60				1.0624
		75	1.0332			1.0372
0.120"	0.016"	1	27.5262	21.9635		
		2	13.5989	12.2268		
		5	5.6827	5.4571	1.4720	
		10	3.1934	3.0236	1.3654	
		15	2.1497	2.1989	1.3062	
		25	1.5495	1.5615	1.2340	1.2947
		40	1.2442	1.2362	1.1698	1.1600
		50	1.1524	1.1399	1.1393	1.1145
		60				1.0830
		75	1.0442		1.0294	1.0494
0.120"	0.020"	1	34.1078	27.1649		
		2	16.7248	15.0125		
		5	6.6163	6.5630	1.5892	
		10	3.4356	3.5258	1.4560	
		15	2.4349	2.4964	1.3822	
		25	1.6859	1.7008	1.2921	1.3678
		40	1.3048	1.2948	1.2119	1.1997
		50	1.1903	1.1828	1.1738	1.1429
		60				1.1036
		75	1.0552		1.0720	1.0617

TABLE 2

CELL WITH 80 MESH SCREEN ELECTRODES

Wire diameter: 0.0047"; **Sieve Opening:** 0.0070"

$C = 0.00235$ in.
$T = c/d = 0.401$
% Area Contact: 12.1%

Cell Thickness h inches	Resistance Ratio R_{eff}/R
.01	3.6913
.02	2.3456
.03	1.8970
.04	1.6728
.05	1.5382
.06	1.4480
.07	1.3844
.08	1.3363
.09	1.2990
.10	1.2691
.11	1.2309
.12	1.2242

Values of the resistance ratio, R_{eff}/R were computed (see Table 1) for the four electrode configurations over the following range for each of the parameters:

Cell thickness, H: 0.040", 0.080" and 0.120"
Electrode dimension, c: 0.004", 0.008", 0.010"
0.012", 0.016", 0.020"
% Area contact for square contacts: 1, 2, 5, 10, 15, 25, 40, 50, 75%
% Contact for circular contacts same as for square (except no 75%)*
% Contact for square holes: 5, 10, 15, 25, 40, 50, 75%
% Contact for circular holes:* 25, 40, 50, 60, 75%

In addition, the circular hole configuration was extended to the range of parameters which approximate a porous plate of interest in fuel cells (see Table 2):

Dimension: $c = 4 \times 10^{-5}$ inches (corresponding to 2 dia pores)
60×10^{-5} inches (corresponding to 30 dia pores)
% contact: 25, 40, 50, 60, 75.

In addition, as a special typical case, a cell with 80 mesh (U. S. series) screen contact electrodes with assumed similar contacts, was calculated in Table 3 for several cell thicknesses.

Figures 4, 5, 6, and 7 show plots of some of the data for demonstrating significant relationships within the parameters involved.

IV. DISCUSSION

Exact solutions of the Laplace equation for the four cases considered were possible. Due to slow convergence, however, it was found desirable to carry out the computations to a 100 terms for some cases. In the solution for the resistance ratio, R_{eff}/R, the oscillation of the first (single) summation term (see for instance equation 11), is larger than for the double summation term and, furthermore, it is considerable for small values of the dimensional parameter r. Figure 3 shows that even for a relatively large value of r ($r = 0.1$), it takes at least 25 terms to compute a reliable value for the single summation term. For a case in which $r = 0.01$, it takes about 100 terms to obtain a dependable value of the sum.

Some selected results of Table 1 are presented in Figure 4 for cells of electrolyte thickness $h = 0.040"$ for the four types of electrode contact configurations. From the point of view of access of fuel cell reactants, the percentage area of contact is an important consideration. As seen in Figure 4, the resistance ratio values are reasonably close for either square and circular contacts or for square and circular holes; however, below a 40% area contact, at any given value of fractional area contact, it makes a considerable difference whether the contacts are made with perforated plates or with screens. Thus, for example for the parameters presented in Figure 4, at a 20% area contact, the screens give a resistance ratio of 2.5, while the perforated plate, only 1.5. Such a difference would not normally be intuitively anticipated.

As the cell thickness increases, the resistance ratio for the case of discontinuous contact electrodes becomes smaller, as should be obvious by inspection of the equations. Thus, in Figure 5, for cells twice as thick as those of Figure 4, (i.e., $h = 0.080"$), much smaller resistance ratios are evident. Again here, for the case of perforated holes, the resistance ratio, R_{eff}/R, values are lower than for the case of square or circular contacts (i.e., screen electrodes).

To illustrate the importance of the dimensional parameter "c", its affect on the resistance ratio is shown in Figure 6 for a thin cell ($h = 0.040"$) with a deliberately low contact area of only 1%. By reason of physical dimensions (very low mesh screens), this case, of little practical value, serves only to emphasize the point that all things being equal, dimensional parameter "c" should be made, whenever possible, very small. Thus, for a given percentage area contact and given thickness, it is desirable to have as much distributed contact as possible (i.e., small values of dimension "c"). For the more practical case, illustrated in Figure 7, with a 40% area contact and a cell thickness of 0.040", it can be seen that decreasing parameter "c" from 0.020 to 0.010 results in a lowering of the resistance ratio from about 1.9 to about 1.45 for the case of square or round contacts.

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*Lower and upper limits set by nature of configuration.

TABLE 3A

CELLS WITH POROUS PLATES

With 2 Micron Dia. Pores ($C = 0.00004''$)

Cell Thickness h, inches	% Area Contact	Resistance Ratio R_{eff}/R
0.040"	25%	1.00387
	40	1.00210
	50	1.00151
	60	1.00109
	75	1.00065
0.080"	25%	1.00111
	40%	1.00060
	50	1.00043
	60	1.00031
	75	1.00018
0.120"	25%	1.00073
	40	1.00039
	50	1.00028
	60	1.00021
	75	1.00012

TABLE 3B

With 30 Micron Dia. Pores ($C = 0.00060''$)

Cell Thickness h, inches	%Area Contact	Resistance Ratio R_{eff}/R
0.040"	25%	1.03322
	40	1.01804
	50	1.01290
	60	1.00935
	75	1.00557
0.080"	25%	1.01661
	40	1.00902
	50	1.00645
	60	1.00468
	75	1.00279
0.120"	25%	1.01107
	40	1.00608
	50	1.00430
	60	1.00312
	75	1.00185

To bring the results of these computations closer to the realm of practical applications, resistance ratio values have been computed for cells of various thicknesses, employing 80 mesh (U. S. series) screen electrodes. From the data for such screens, it can be calculated that the percentage area contact is 12.10%. Table 2 gives the results of these computations. The results indicate that for a cell thickness of 0.040", the effective resistance is 67% higher due to the screen electrodes as compared with solid contact electrodes. And even for a cell three times as thick (h = 0.120"), the resistance penalty is still more than 22%. Thus, the consideration of the geometric parameters in a matrix electrolyte cell are indeed important.

Finally, it was of some interest to consider an idealized form of a porous electrode. Porous electrodes are now commonly employed in fuel cell technology. Contacts with the electrolyte are usually made with plates having pore diameter either in the range of 1-4 microns or in the range of 20-40 microns. If for simplicity, the porous plate can be treated as one perforated with uniform round holes located on square centers, then equation (17) can be employed. Results of these computations are given in Tables 3A and 3B for 2 micron and 30 micron pore diameter plates, respectively. The variations of cell thickness and percent area contact in the range of 25 - 75% are also included. The immediately obvious conclusion is that the cell resistance with porous plates is significantly lower than for any perforated plates or screens considered in this study. The highest value of 1.033 is shown in Table 3B for the coarse plate and the smallest value of cell thickness and percent area contact. This indicates that at least from the point of view of primary current distribution and resistance ratio, the use of even coarse porous plates is advantageous over other discontinuous contact electrodes. The consequences of such considerations in the basic design of electrochemical fuel cells is fairly obvious. In the final analysis, however, such considerations must be linked with the mass transport aspects of the reactants and products to and from the contact zone between electrode and electrolyte.

ACKNOWLEDGMENTS

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V. NOMENCLATURE

A	- General area portion (conducting or hole) of the electrode area element 2d x 2d (see Figure 2).
A_c	- Conducting portion of area element 2d x 2d.
c	- Half of diameter or side of contact or hole area portion A.
d	- Half of the center distance between conducting portions.
h	- Thickness of electrolyte matrix, cm or in.
i	- Current density, amp/cm^2
K	- $i\rho/(A/4d^2)$
R	- Total resistance of a cell with solid continuous electrodes, ohms.
R_{eff}	- Total resistance of a cell with discontinuous electrodes, ohms.
r	- c/d
V	- Potential, volts.
x, y, z	- Cartesian coordinates in electrolyte matrix geometry.
ρ	- Specific resistivity of electrolyte matrix, ohm - cm.

VI. LITERATURE CITED

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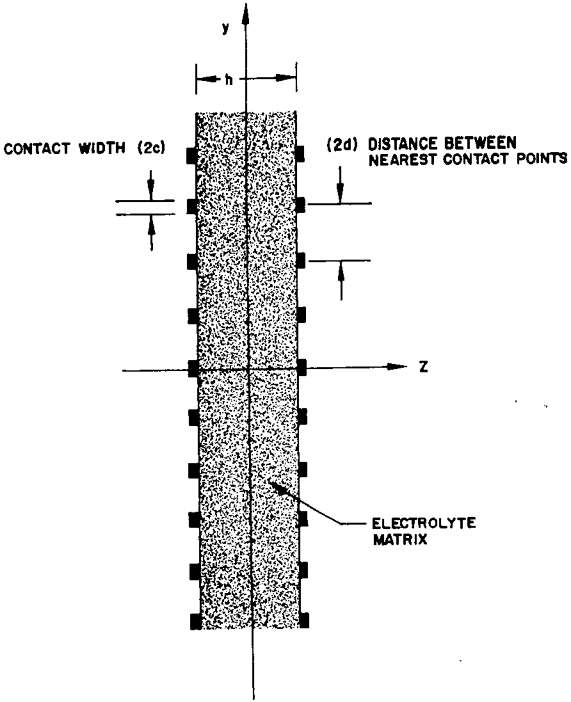

DIMENSIONS c AND d ARE THE SAME IN THE X-DIRECTION

FIG. I CROSSECTION OF A MATRIX CELL WITH DISCONTINUOUS ELECTRODE CONTACTS

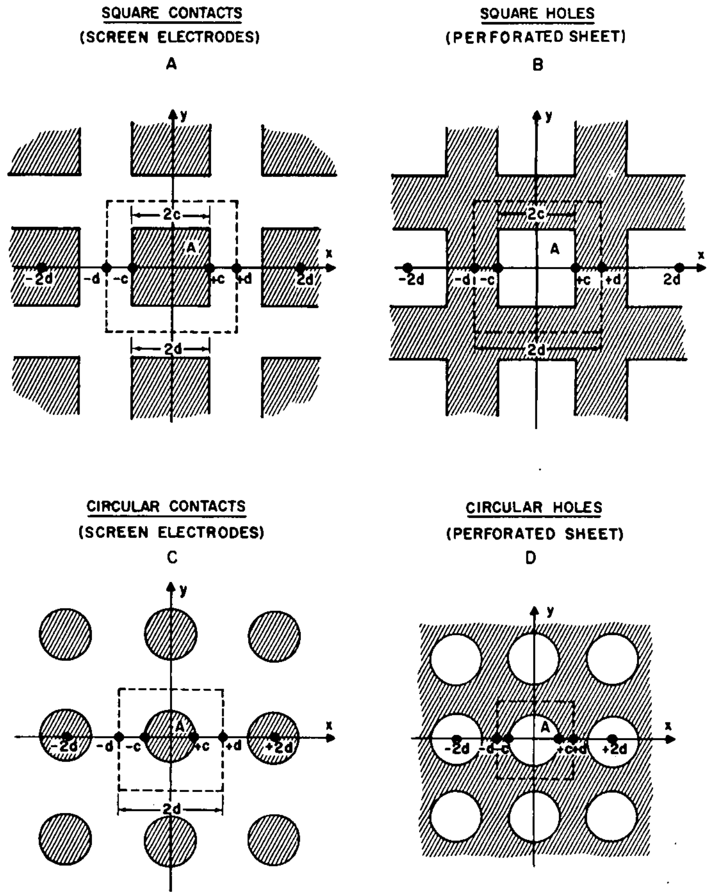

FIG. 2 PRESENTATION OF CONDUCTING OR HOLE AREA PORTION "A" WITHIN SQUARE 2d x 2d.

VII. APPENDIX

Detailed derivation for the expression for resistance ratio R_{eff}/R for the square contact case is included in this section. The derivation of this ratio for the other three cases was arrived at in a similar fashion.

From the previous section:

$$C_1 = \frac{Kc^2}{d^2} \tag{10}$$

The result of integrating the right hand side of equation (8) is:

$$Kc\,\frac{d}{n\pi}\sin\frac{n\pi c}{d}\ ;\ \text{or}\ Kc\,\frac{d}{m\pi}\sin\frac{m\pi c}{d}$$

The result of integrating the right hand side of equation (9) is:

$$K\,\frac{d^2}{nm\pi^2}\sin\frac{n\pi c}{d}\sin\frac{m\pi c}{d}$$

and substitution of equations (5), (8), (9), and (10) into equation (3) yields the general solution for the potential distribution for either square holes or square contacts:

$$\begin{aligned} V = C_0 + K \Bigg\{ & \frac{c^2}{d^2} + \frac{2c}{\pi^2}\sum_{n=1}^{\infty}\sin\frac{n\pi c}{d}\cos\frac{n\pi x}{d} \\ & \cdot \sinh\frac{n\pi z}{d}\Big/ n^2\cosh\frac{n\pi h}{2d} + \frac{2c}{\pi^2}\sum_{m=1}^{\infty}\sin\frac{m\pi c}{d} \\ & \cdot \cos\frac{m\pi y}{d}\sinh\frac{m\pi z}{d}\Big/ m^2\cosh\frac{m\pi h}{2d} \\ & + \frac{4d}{\pi^3}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sin\frac{n\pi c}{d}\sin\frac{m\pi c}{d}\cos\frac{n\pi x}{d}\cos\frac{m\pi y}{d} \\ & \sinh\frac{\sqrt{n^2+m^2}\,\pi z}{d}\Big/\left(nm\sqrt{n^2+m^2}\cosh\frac{\sqrt{n^2+m^2}\,\pi h}{2d}\right) \end{aligned} \tag{10a}$$

For squares, the value of K in equation (2) is given by:

$$K = \frac{i\rho}{r^2} \tag{10b}$$

and substituting for K into equation (10a)

$$\begin{aligned} V = C_0 + i\rho z & + \frac{2i\rho d}{\pi^2 r}\sum_{n=1}^{\infty}\frac{\sin n\pi r\,\sinh\left(\frac{n\pi z}{d}\right)}{n^2\cosh\left(\frac{n\pi h}{2d}\right)} \\ & \cdot \cos\frac{n\pi x}{d} + \frac{2i\rho d}{\pi^2 r}\sum_{m=1}^{\infty}\frac{\sin m\pi r\,\sinh\left(\frac{n\pi z}{d}\right)\cos\frac{m\pi y}{d}}{m^2\cosh\left(\frac{m\pi h}{2d}\right)} \\ & + \frac{4i\rho d}{\pi^3 r^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\sin n\pi r\,\sin m\pi r\,\sinh\left(\frac{\sqrt{n^2+m^2}\,\pi z}{d}\right)}{mn\sqrt{n^2+m^2}\cosh\left(\frac{\sqrt{n^2+m^2}\,\pi h}{2d}\right)} \\ & \cdot \cos\frac{n\pi x}{d}\cos\frac{m\pi y}{d} \end{aligned} \tag{10c}$$

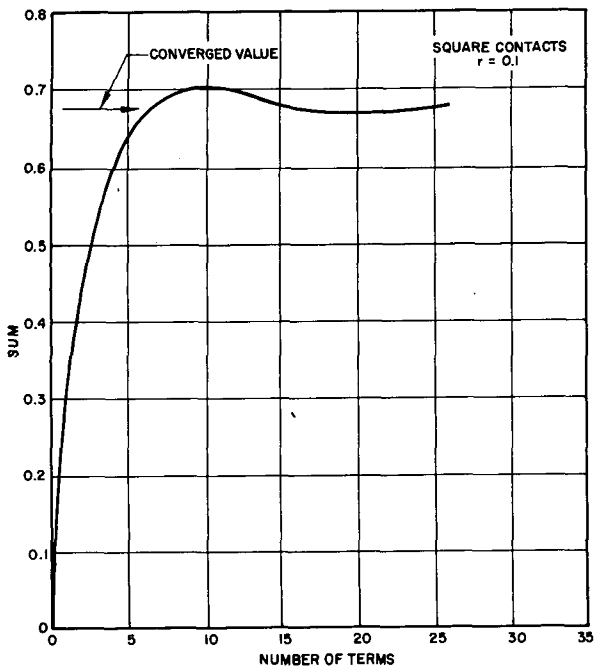

FIG 3. CONVERGENCE CHARACTERISTICS FOR SINGLE SUMMATION TERM IN EQUATION (11)

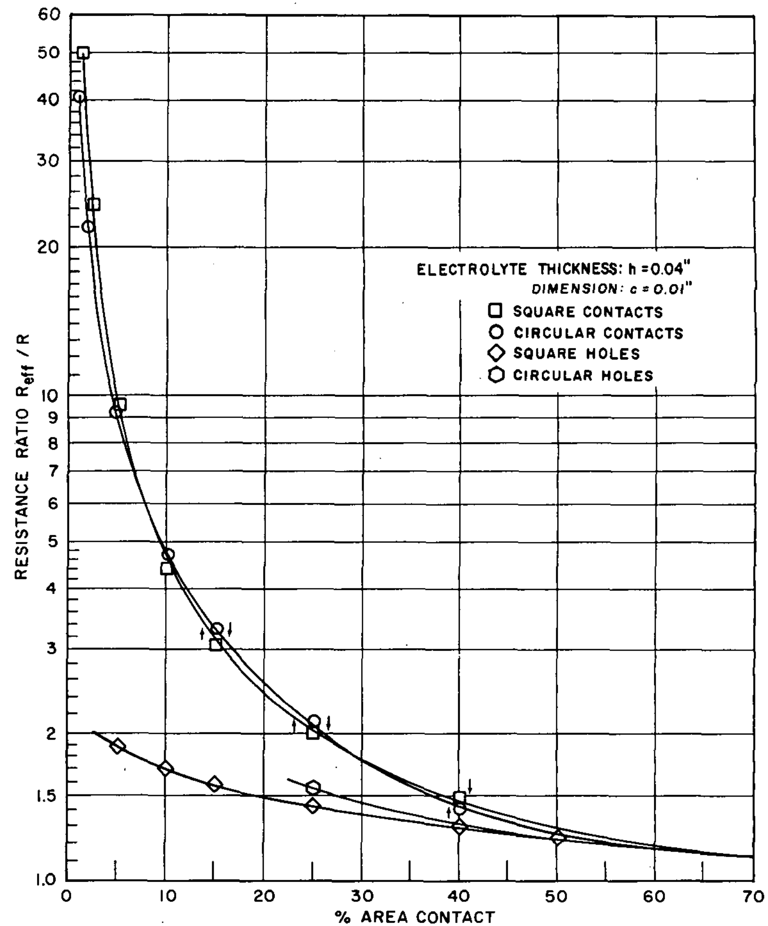

FIG. 4 RESISTANCE RATIOS AS FUNCTION OF PERCENTAGE CONTACT AREA FOR VARIOUS ELECTRODE CONFIGURATIONS

Using this expression for V, the ratio of the resistance of an electrode pair consisting of square contacts to that of a pair of plane electrodes is given by:

$$\frac{R_{eff}}{R} = \frac{V(x = o,\ y = o,\ z = \frac{h}{2}) - V(x = o,\ y = o,\ z = \frac{-h}{2})}{i\ \rho\ h}$$

$$= \frac{i\ \rho\ h}{i\ \rho\ h} + \frac{8d}{\pi^2 h r} \sum_{n=1}^{\infty} \frac{\sin n\pi r}{n^2} \tanh \frac{n\pi h}{2d} \tag{11}$$

$$+ \frac{8d}{\pi^3 r^2 h} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\sin m\pi r \sin n\pi r}{n\ m \sqrt{n^2 + m^2}} \tanh \frac{\sqrt{n^2+m^2}\,\pi h}{2d}$$

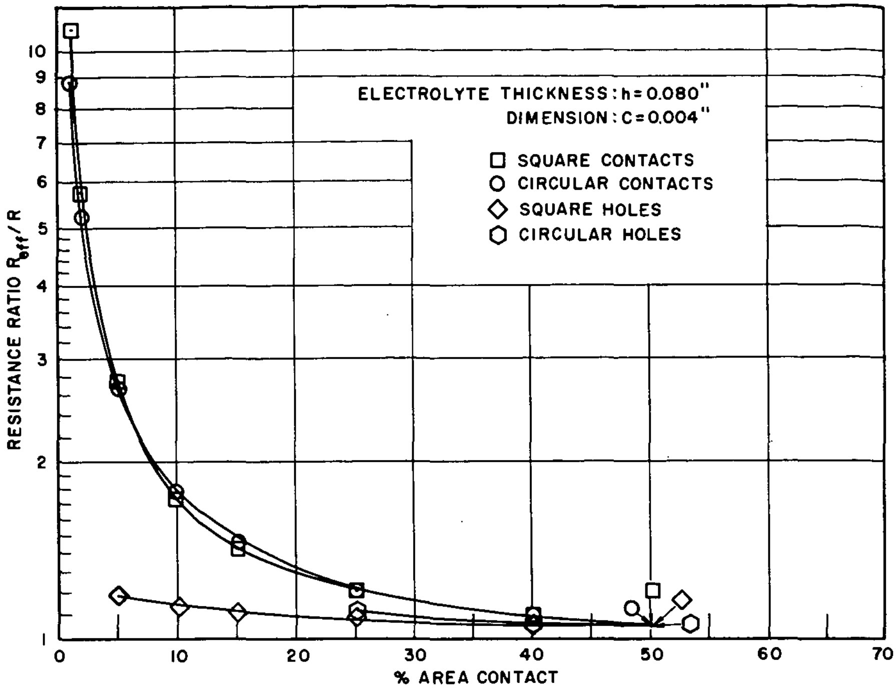

FIG. 5 THE EFFECT OF PERCENT AREA CONTACT ON THE RESISTANCE RATIO R_{eff}/R

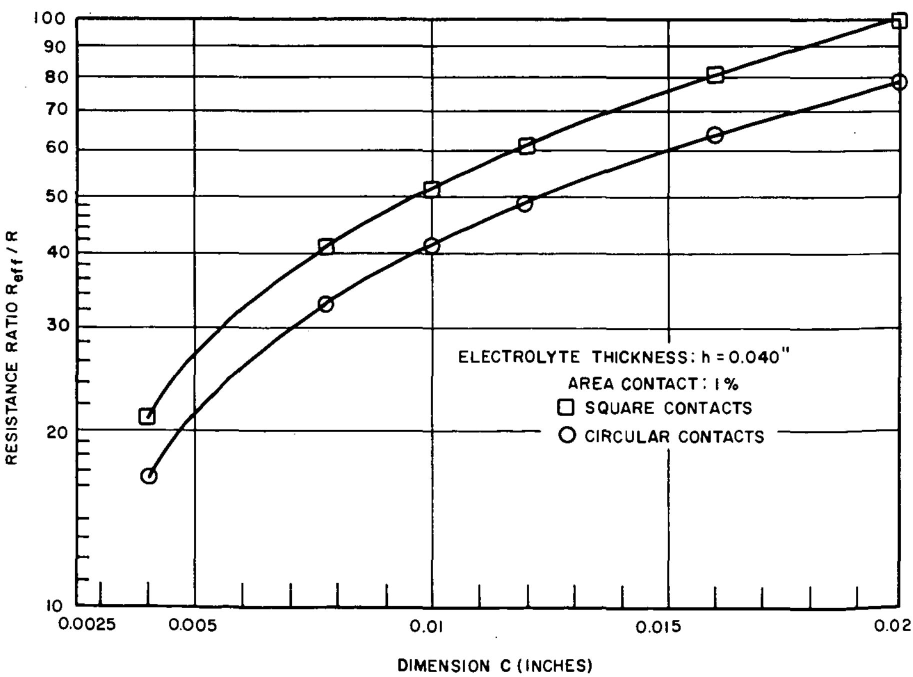

FIG. 6 THE EFFECT OF THE DIMENSION "C" ON THE RESISTANCE RATIO R_{eff}/R AT 1% AREA CONTACT

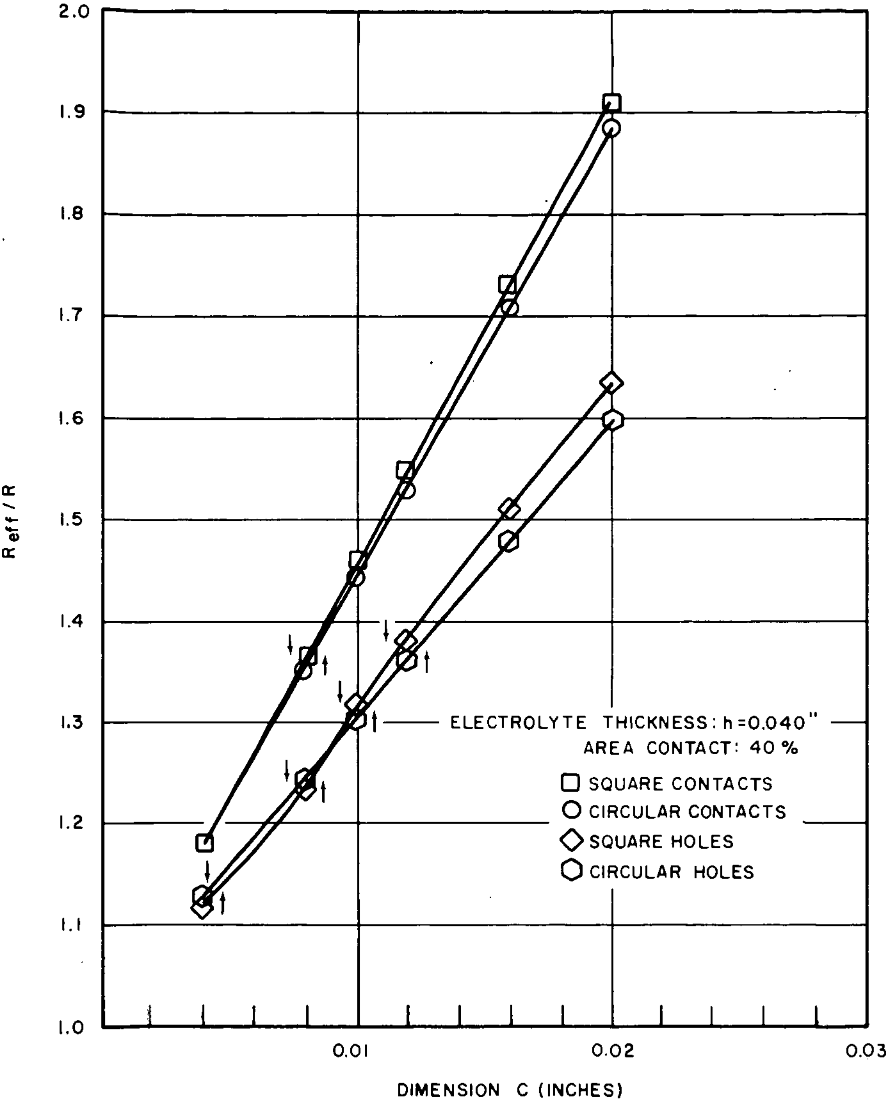

FIG. 7 THE EFFECT OF THE DIMENSION "C" ON THE RESISTANCE RATIO R_{eff}/R AT 40% AREA CONTACT