The influence of particle size distribution on the combustion rates in a batch fed fluidised bed

E.S. Garbett and A.B. Hedley

Sheffield Coal Research Unit, Department of Chemical Engineering and Fuel Technology, University of Sheffield, Sheffield, S1 3JD United Kingdom.

Introduction

The advantages of burning coal in fluidised bed combustors (FBC) have been repeatedly demonstrated over the past few years and commercial units are now being built and supplied to industry (1). The design of such FBC's is based on the practical experience gained from pilot plant studies and also on the predictions of mathematical models. Central to any model of coal combustion in FBC's is the rate at which particles burn in the bed which determines the carbon loading and hence pollutant emissions, heat release rates, coal feed rates and elutriation rates (2). Particle size and temperature usually indicate the controlling mechanism of combustion; large particles and high temperature suggest that diffusion is dominant whereas chemical kinetics became important for small particles and low temperatures (3). In a typical FBC the particle sizes range from that of the maximum of the input feed down to that of particles which are about to be elutriated and particle temperatures can have a value anywhere between the bed temperature T_b and T_b + 200K (4,5,6). It is not unreasonable to assume therefore, that the combustion of particles can be controlled by a combination of diffusional and chemical processes acting simultaneously. Indeed, the data of Avedesian and Davidson (7), where the combustion of char particles in a batch fed fluidised bed was assumed to be diffusion controlled, has recently (6) been shown to be consistent with the alternative situation described above where both processes are acting together. The effect on the carbon loading in an FBC where the combustion is either controlled by diffusion or by a combination process has been investigated by Garbett and Hedley (8) who also showed the importance of particle surface temperatures. The particle size distribution of the coal feed is a parameter which is important in the design of FBC's but whose influence on the combustion rates in the bed has not been studied. It is the purpose of this paper therefore, to develop a theory to predict the burnaway, burnout time and particulate phase oxygen concentration in a batch fed FBC and show their dependence on the input particle size distribution which is accurately known.

The Model

The model of the batch fed FBC employed here is essentially the two-phase bubbling bed model of Avedesian and Davidson (7) where a bed of inert particles at a temperature T_b is fluidised by air at a superficial velocity U_o and a minimum fluidised velocity U_{mf}. The oxygen required for combustion which is controlled by a combination of diffusion and chemical kinetics, is transferred to the char particles in the particulate phase from the bubbles and the reaction is given by $C + O_2 \rightarrow CO_2$. The batch is assumed to comprise solely of char particles so that devolatilisation is neglected. It is also assumed that the particles do not swell or break up when introduced to the bed and so retain their original size distribution.

The Theory

The reaction rate of a single particle of char of mass m_p in a fluidised bed where both diffusional and chemical kinetics are considered simultaneously, can be expressed in terms of diffusional and chemical resistances as follows (3).

$$R = \frac{K_c K_D C_p}{K_c + K_D} \qquad 1)$$

where $K_D = -k_1 x$ 2)

$K_c = -k_2 x^2$ 3)

(See equations 28) and 31) for values of k_1 and k_2)) C_p is the oxygen concentration (moles/m^3) in the particulate phase of the bed and is a function of time.

The rate of consumption of carbon at the particle surface is therefore given by

$$\frac{dm_p}{dt} = \frac{-k_1 k_2 x_p^3 \; M_o \; C_p}{k_1 x_p + k_2 x_p^2} \qquad 4)$$

leading to

$$\frac{dx_p}{dt} = \frac{-2M_o C_p}{\pi\rho} \left[\frac{1}{\frac{x_p}{k_1} + \frac{1}{k_2}} \right] \qquad 5)$$

where x_p is the diameter of a particle of mass m_p and M_o is the molecular weight of oxygen.

For convenience let $F(C) = \frac{2M_o C_p}{\pi\rho}$, $e = \frac{1}{k_2}$ and $f = \frac{1}{k_1}$

If after a time t a particle whose original diameter was x burns to give a diameter x_t, then

$$x_t = \frac{1}{f} \left[(e^2 + 2efx + f^2x^2 - 2f \int_o^t F(C)dt)^{\frac{1}{2}} - e \right] \qquad 6)$$

Let the original size distribution of a batch of carbon delivered to the fluidised bed be given by an equation of the form.

$$W_i = \theta(x) \qquad 7)$$

where W_i is the initial weight of the batch which consists of particles whose diameter is greater than or equal to x. It will be assumed that x must lie in the range $x_1 \leq x \leq x_2$ such that whatever the form of the function θ the conditions $\theta(x_1) = 1$ and $\theta(x_2) = 0$ are satisfied.

The initial mass of the batch which has particles whose diameters lie between x and x +dx is therefore, $m_i \; dW_i$, where m_i is the mass of the original batch. After a time t the mass will reduce to $m_i \; dW_i \; x_t^3/x^3$.

Therefore the mass of carbon in the bed after a time t is

$$m = \int_{x_1}^{x_2} m_i \; \frac{x_t^3}{x^3} \; \psi(x) \; dx \qquad 8)$$

where $\psi(x)\,dx = -dW_i$ and the lower limits of the integral are chosen to indicate

all the initial values of x for which $x_t > o$. Suppose an initial diameter y_o is defined so that all particles with an initial diameter $x < y_o$ would have burned away completely after a time t, then the mass fraction of carbon in the bed at this time t is

$$\frac{m}{m_i} = \int_{x=z}^{x=x_2} \frac{x^3_t}{x^3} \psi(x)\,dx \qquad 9)$$

where $z = x_1$ if $y_o \leq x_1$

and $z = y_o$ if $y_o > x_1$

By substituting y_o in equation 6) we have

$$\int_o^t F(C)\,dt = ey_o + \frac{f}{2} y_o^2 \qquad 10)$$

Combining equations 6), 8) and 10) gives

$$\frac{m}{m_i} = \int_{x=z}^{x=x_2} \frac{\left\{\left[\left(\frac{k_1}{k_2}\right)^2 + 2\left(\frac{k_1}{k_2}\right)(x-y_o)+(x^2-y_o^2)\right]^{\frac{1}{2}} - \frac{k_1}{k_2}\right\}^3 \psi(x)}{x^3}\,dx \qquad 11)$$

Burnout time

To derive an expression for the burn-out time we need to investigate dm/dt or $d(m/m_i)/dt$. From an oxygen balance on the bed (7) we have

$$12(C_o-C_p)A[U_o-U_{mf})(\exp(-B))] = -\frac{dm}{dt} \qquad 12)$$

where C_o is the inlet oxygen concentration (mole/m^3).

Therefore $C_p = C_o + \frac{dm/dt}{F(U_o)}$ 13)

where $F(U_o) = 12A[U_o - (U_o-U_{mf})(\exp(-B))]$

It can therefore be shown that

$$\frac{dm}{dt} = \frac{2C_o}{\left[\frac{\pi\rho}{M_o}\cdot\frac{(e+fy_o)}{dm/dy_o} - \frac{2}{F(U_o)}\right]} \qquad 14)$$

Also, from equation 13)

$$C_p = C_o\left[1 + \frac{1}{\left(\frac{F(U_o)\pi\rho(e+fy_o)}{2M_o\,dm/dy_o}\right)-1}\right] \qquad 15)$$

dm/dy_o can be found using the mathematical identity,

$$\frac{d}{dy_o}\left[\int_a^b g(x,y_o)\,dx\right] = \int_a^b \frac{\partial}{\partial y_o}\left(g(x,y_o)\,dx\right) + g(b,y_o)\frac{db}{dy_o} - g(a,y_o)\frac{da}{dy_o} \qquad 16)$$

Applying this to equation 11) gives

$$\frac{dm}{dy_o} = -3k_1(e+fy_o)m_i \int_{x=z}^{x=x_2} F_1(x,y_o)\ \psi(x)\,dx \qquad 17)$$

where

$$F_1(x,y_o) = \frac{\left\{\left[\left(\frac{k_1}{k_2}\right)^2 + 2\left(\frac{k_1}{k_2}\right)(x-y_o) + (x^2-y_o^2)\right]^{\frac{1}{2}} - \frac{k_1}{k_2}\right\}^2}{x^3\left[\left(\frac{k_1}{k_2}\right)^2 + 2\left(\frac{k_1}{k_2}\right)\cdot(x-y_o) + (x^2-y_o^2)\right]} \qquad 18)$$

Substituting dm/dy_o from equation 17) into equation 14)

re-arranging and integrating the result gives

$$\int_o^t dt = -\frac{1}{F(U_o)C_o}\int_{m_i}^{m} dm - \frac{\pi\rho}{6M_oC_ok_1m_i}\cdot\int_{m_i}^{m}\frac{dm}{\int_z^{x_2} F_1(x,y_o)\psi(x)\,dx} \qquad 19)$$

It can be shown that equation 19) reduces to

$$t = \frac{m_i\ (1-m/m_i)}{F(U_o)C_o} + \frac{\pi\rho\ y_o^2}{128C_ok_1} + \frac{\pi\rho\ y_o}{64C_ok_2} \qquad 20)$$

<u>The form of the initial size distribution</u>

Assume that the original carbon sample before any sieving into limited size range fractions, can be described by a Rosin-Rammler distribution

$$W = \exp\ (-bx^n) \qquad 21)$$

where b and n are constants. If the original material is then sieved into limited size fractions and only particles with diameters in the range $x_1<x<x_2$ are retained then the weight fraction W_i of the sieved material is (9)

$$W_i = \frac{\exp(-bx^n) - \exp(-bx_2^n)}{\exp(-bx_1^n)-\exp(-bx_2^n)}$$

Therefore

$$\psi(x) = \frac{bnx^{n-1}\ \exp(-bx^n)}{\exp(-bx_1^n)-\exp(-bx_2^n)} \qquad 22)$$

and the initial weighted mean particle size $\bar{x}_i$ is (9)

$$\bar{x}_i = \frac{\sqrt{(\frac{1}{n}+1)}}{b^{1/n}} \qquad 23)$$

Defining dimensionless parameters $X = \frac{x}{\bar{x}_i}$ and $Y = \frac{y_o}{\bar{x}_i}$ then from equation 22) we get

$$\psi(\bar{x}_i, x) = \frac{naX^{n-1}\exp(-aX^n)}{\exp(-aX_1{}^n)-\exp(-aX_2{}^n)} \qquad 24)$$

where $a = \left[(\frac{1}{n}+1) \right]^n$

Substitution of $\psi(\bar{x}_i, x)$ from equation 24) into equation 11) and making other appropriate terms dimensionless gives

$$\frac{m}{m_i} = \frac{1}{N}\int_Z^{X_2} \left\{ F_2(\bar{x}_i, X, Y) - \left(\frac{k_1}{k_2\bar{x}_i}\right)\right\}^3 an\, X^{n-4}\, e^{-ax^n}\, dx \qquad 25)$$

where

$$F_2(\bar{x}_i, X, Y) = \left[\left(\frac{k_1}{k_2\bar{x}_i}\right)^2 + \left(\frac{2k_1}{k_2\,\bar{x}_i}\right)(X-Y) + (X^2-Y^2)\right]^{\frac{1}{2}}$$

$$N = \exp(-aX_1{}^n) - \exp(-aX_2{}^n) \quad \text{and} \quad Z = \frac{z}{\bar{x}_i}$$

The lower limit of integration in equation 25) is given by

$Z = X_1$ if $Y \leqslant X_1$

and

$Z = Y$ if $Y > X_1$

Similarly equation 20) for the burn-out time, becomes

$$t = \frac{m_i\,(1-{}^m/m_i)}{F(U_o)C_o} + \frac{\pi\rho(\bar{x}_i{}^2Y^2)}{128C_o k_1} + \frac{\pi\rho(\bar{x}_i Y)}{64C_o\, k_2} \qquad 26)$$

and equation 15), the particulate phase oxygen concentration, becomes

$$C_p = C_o\left[1 - \frac{1}{\left(\frac{F(U_o)\,\pi\rho}{192k_1 m_i \int_Z^{X_2} F_3(\bar{x}_i, X, Y)\,d}\right) + 1}\right] \qquad 27)$$

where

$$\int_Z^{X_2} F_3(\bar{x}, X, Y)\,dX = \frac{na}{N\bar{x}_i{}^{3/2}} \int_Z^{X_2} \left[F_2(\bar{x}_i, X, Y)\left(\frac{k_1}{k_2\bar{x}_i}\right)\right]^2 X^{n-4}\, e^{-aX^n}\, dX$$

Numerical Calculations

The diffusion constant k_1 from equation 2) can be expressed as follows (7)

$$k_1 = 2\pi k D_E \qquad 28)$$

where k is a constant relating the mass of carbon consumed to the mass of O_2 transported to the particle surface. It is assumed here that the carbon-oxygen reaction can be represented by $C + O_2 \rightarrow CO_2$ so that k = 12/32

D_E is the effective diffusion coefficient and can be represented by (7)

$$D_E = \frac{D_G \, Sh}{2} \qquad 29)$$

where Sh is the local particle Sherwood Number and D_G is the diffusion coefficient calculated from (3)

$$D_G = D_i \frac{(P_i)}{P} \left(\frac{T}{T_i}\right)^{1.75} \qquad 30)$$

The chemical constant k_2 can be represented by (10,11)

$$k_2 = k' \, T_b^{\frac{1}{2}} \exp(-E/R_g T_p) \qquad 31)$$

The above data is summarised in Table 1. The bed data used in the calculations are that of Avedesian and Davidson (7).

Table 1

Data used in calculations

Symbol	Value	Reference
k	0.375	
T_i	1000 K	(3)
D_i	1.61	(3)
T_b	1173 K	(7)
T_p	1173 K	(7)
k´	1034	(11,12)
Sh	1.8	(8)
E	15 K cal/mole K	
P/P_i	1	

The three distributions used in the calculations are summarised in Table 2. In all the results presented here the top and bottom size of each distribution were constant at 3 and 0.3mm respectively.

Table 2

Particle size data

	b=11.12	b=625	b=10000
n	1.5	4	4
$\bar{x}_i$	0.1813	0.1813mm	0.0906mm
Γ	0.9033	0.9064	0.9064
x_1	0.3mm	0.3mm	0.3mm
x_2	3mm	3mm	3mm
m_i	5gm	5gm	5gm

Results and Discussion

Evaluation of m/m_i

To demonstrate the type of results to be expected from the theory equation 25) was numerically integrated to give values of m/m_i for increasing values of Y using the values of k_1 and k_2 indicated in Table 1 and the distributions in Table 2. For comparison the values of m/m_i for the pure diffusional and chemical cases were calculated using a method similar to that described by Leesley and Siddall (9) for pulverised fuel. Figures 1, 2 and 3 show the variation of m/m_i, the unburnt fraction of carbon remaining, with the dimensionless particle diameter Y for the three original distributions. In every case the combined curve m_{C+D}/m_i falls inside the envelope of the two extreme conditions of pure diffusional (m_D/m_i) and of pure chemical (m_C/m_i). The curves represented in these figures (1,2, and 3) are not burnaway rates but indicate the changing particle size distribution as the batch disappears. For example, in figure 1, when 50% of the batch has burnt away (i.e. $m/m_i = 0.5$) the combined case requires that all particles of a size below 18% of the maximum particle size in the original batch must have disappeared. Corresponding values for the pure diffusion case and pure chemical case are 34% and 12% respectively. Thus for a given carbon loading the carbon particle size distribution in the bed will be different for different combustion mechanisms and this will obviously influence important phenomena such as elutriation, and NO reduction by char. To combine Figures 1 to 3 for comparison purposes the data have been represented in Figure 4 as a ratio of unburnt fraction for the combined case (m_{C+D}) to that for the diffusion case (m_D) as a function of the particle diameter Y. It is clearly seen that as the original size distribution moves from a wide one (n=1.5, b=11.12) to a fine one (n=4.0, b=10,000) the difference between m_{C+D} and m_D increases

Evaluation of t

The burnaway rates of the batch for each original size distribution and for a combustion mechanism where both diffsuion and chemical kinetics are acting simoultaneously were calculated from equation 26) and are shown in Figure 5. The burnaway rate is higher for the distribution represented by n=4.0, b=10,000 than for the other two distributions as would be expected. What is not evident

from Figure 5, due to the scale, is that the time for total burnaway is the same for each distribution since they all have the same maximum particle size of 3mm.

Evaluation of C_p

Equation 27) was numericatly integrated to give the particulate oxygen concentration C_p for the three distributions. The results are shown in Figure 6. An interesting feature of these results is the value of C_p at t=o. According to Avedesian and Davidson (7) the value of C_p should be almost zero for the batch fed system but it is clear from Figure 6 that this is not the case.

Equations 25), 26) and 27) are valid only for constant temperature and pressure. These conditions are usually met in FBC's except that the particle temperature T_p can vary during burnaway and can be appreciably higher than the bed temperature (4,5,6). The effect of increasing T_p would be to increase the value of k_2 relative to k_1 and the controlling mechanism would tend towards that of diffusion. Increasing the pressure of the system would only have a significant effect if the combustion rate was initially dominated by chemical kinetics.

Conclusions

A theoretical model has been developed which predicts the change in size distribution during burnaway, burn-out times and particulate phase oxygen concentrations as a function of original particle size distribution in a batch fed fluidised bed. If the original size distribution is accurately known a batch fed experiment could be devised to determine the role of chemical kinetics for a given type of coal and would thus be an aid to modelling of fluidised bed combustion.

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Nomenclature

a	Constant defined by equation 24)
A	Area of fluidised bed (m^2)
b	Rosin-Rammler constant - equation 21)
B	Oxygen exchange parameter - equation 12)
C_o	Oxygen concentration of fluidising air ($mole/m^3$)
C_p	Particulate phase oxygen concentration ($mole/m^3$)
D_G	Gas diffusion coefficient (m^2/s)
e	constant ($= 1/k_2$)
f	constant ($= 1/k_1$)
$F(U_o)$	Defined by equation 12)
F(C)	Function of Oxygen concentration ($= 2M_oC_p/\pi\rho$)

k_1	Constant defined by equation 2)	
k_2	Constant defined by equation 3)	
k'	Constant defined by equation 31)	
K_c	Chemical rate constant	
K_D	Diffusional rate constant	
m	Mass of char in bed at time t	(kg)
m_i	Initial mass of batch	(kg)
m_p	Mass of single char particle	(kg)
N	Function of particle size - equation 25)	
P	Pressure	(atm)
R	Reaction rate of single particle	(kg/s)
R_g	Gas constant	
t	Burnaway time	(s)
T	Temperature	(K)
T_b	Bed temperature	(K)
T_i	Reference temperature - equation 30)	(K)
T_p	Particle surface temperature	(K)
U_{mf}	Minimum fluidising velocity	(m/s)
U_o	Superficial fluidising velocity	(m/s)
W	Weight fraction of coal	(kg)
x	Particle diameter	(mm)
$\bar{x}_i$	Initial weighted mean particle size	(mm)
x_p	Particle diameter	(mm)
x_t	Particle diameter	(mm)
X	Dimensionless particle diameter	$(= x/\bar{x}_i)$
y_o	Particle diameter	(mm)
Y	Dimensionless particle diameter	$(= y_o/\bar{x}_i)$
z	Particle diameter (lower limit of integration)	mm
Z	Dimensionless particle diameter	$(= z/\bar{x}_i)$

Greek Symbols

ρ	Char density
θ	Size distribution function equation 7)
ψ	Size distribution function equation 8)
Γ	Gamma function

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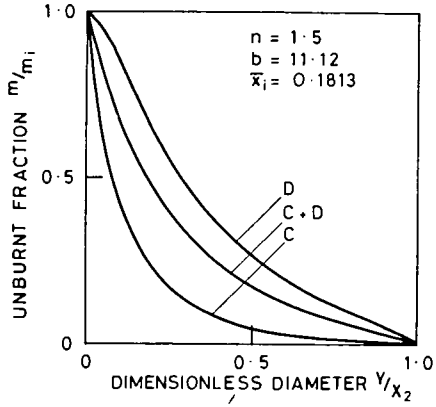

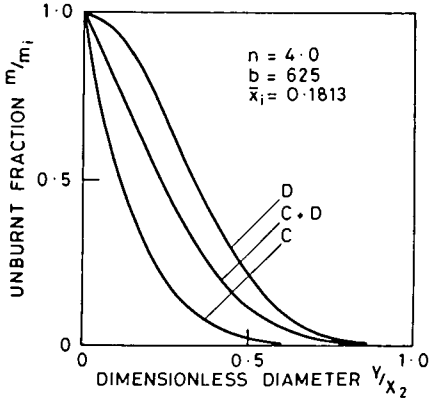

Fig. 1. (top) and Fig. 2. (bottom). Unburnt mass fraction (m/m_i) of the batch during burnaway.

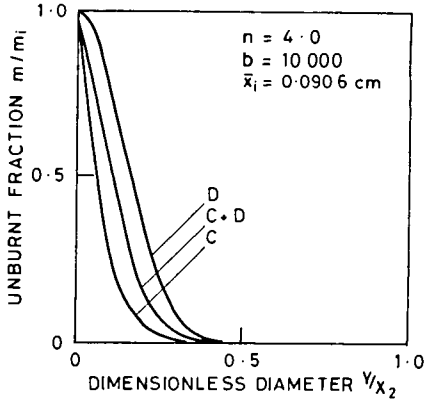

Fig. 3. (top). Unburnt mass fraction (m/m_i) of the batch during burnaway.

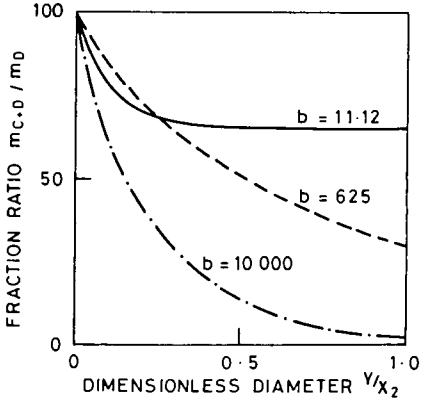

Fig. 4. (bottom). Unburnt mass fraction ratio (m_{C+D}/m_D) during burnaway for the three distributions.

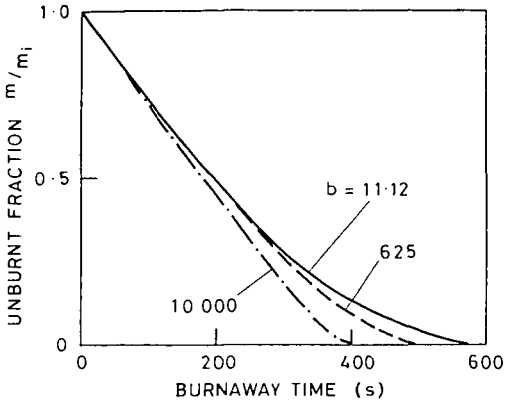

Fig. 5. (top). Burnaway rate of the batch for the three distributions.

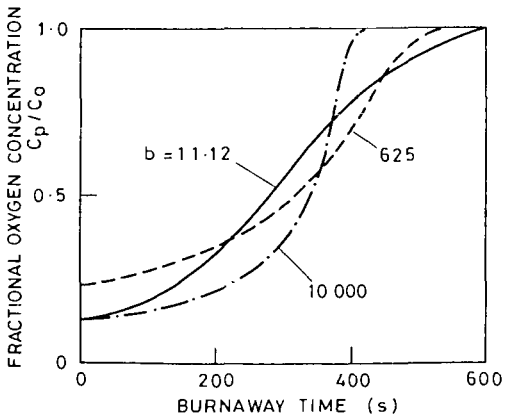

Fig. 6. (bottom). Particulate phase oxygen concentration (C_p) during burnaway for the three distributions.