MATHEMATICAL ANALYSIS OF A MSW ROTARY INCINERATOR

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ABSTRACT

A computerized mathematical model of a refractory-lined rotary kiln combustor with a vertically well-mixed bed has been developed. Bed height varies with horizontal distance as fuel is consumed. The kiln contains a pyrolysis zone, followed by a combustion zone. A kinetic model for cellulose pyrolysis simulates the thermal destruction of MSW in the first zone. The combustion of particles in the second zone follows a shrinking-core model. Model behavior is in good agreement with the performance of commercial units.

INTRODUCTION

A number of the waste-to-energy facilities operating in the United States utilize a rotary kiln combustor as the principal treatment unit in their systems. The barrel of the kiln may be either refractory-lined or water-walled. Waste is fed to the kiln from a receiving system and the products of combustion are treated and released. The combustion gases are polished in a secondary combustor, cleaned of acidic components and particulates, and sent to a stack. Bottom ash is cooled, mixed with fly ash, stabilized and landfilled.

Because of the concerns for full burnout of the combustibles in the feed, the destruction of toxic organics, the segregation and collection of toxic metals, and the efficient collection of energy, developers and operators of MSW kiln combustors must have a thorough understanding of the operation of each element of their waste-to-energy facilities and of the integration of these elements. The mathematical model described in this paper has been constructed and tested to assist in better understanding the operation of the rotary kiln combustor at the heart of many such facilities.

KINETIC MODEL

Incineration is a complex physical and chemical process [1]. Waste fed to the rotary kiln first is demoisturized and heated. When it reaches its reaction onset temperature, pyrolysis, sublimation and other solid volatilization reactions occur under starved-air conditions. After these are complete, the char remaining is oxidized under excess-air conditions. For manageability of the final kiln combustor model developed in this study, a simple kinetic model for cellulose pyrolysis has been adopted to describe the pyrolysis of waste in the starved-air region of the kiln [2]. In addition, the assumption has been made that the surface oxidation of char in the excess-air portion of the kiln is rapid and is largely controlled by the mass transfer of oxygen through the bed.

The kinetic model for the pyrolysis of cellulose is based upon experimental work in the temperature range of 300°C to 1000°C. In its simplest version, the model assumes that cellulose decomposes directly to each reaction product, *i*, by a single, independent reaction pathway,

$$\text{cellulose} \xrightarrow{k_i} \text{product } i$$

and that the kinetics of this process can be modeled by a unimolecular first-order reaction having a rate constant that may be written in the Arrhenius form,

$$\frac{dV_i}{dt} = k_i \, exp\left(\frac{-E_i}{RT}\right)[V_i* - V_i]$$

where V_i is the percent yield of gas *i* at any time t, V_i* is the ultimate (maximum) yield of gas *i*, and k_i and E_i are kinetic parameters. These parameters were correlated with gas-phase pyrolysis products, observed by experimentation, including light hydrocarbons, alcohols, aldehydes, acids, H_2, H_2O, CO, CO_2 and tar.

Two modifications of the kinetic model were required before it could be applied to MSW incineration. First, the rate expressions had to be rewritten so as to account for the decomposition of tar to smaller gaseous species. In order to achieve this, it was assumed that the tar decomposed in the same proportion as the ultimate yields of the various species. A modified ultimate yield was therefore defined according to,

$$V*_{m,i} = V_i* + \frac{V_i*}{\sum_{i=1}^{n} V_i*} V*_{tar}$$

were n is the number of gaseous species evolved during pyrolysis. Secondly, the discrete rate equations were combined, using the modified ultimate yields, to achieve a mass balance.

To reach the char surface and combust the carbon there, the oxygen is assumed to diffuse through the bed to the individual char particles and then through the inert ash layer to the surface of the carbon. The carbon core shrinks as the combustion reaction proceeds. However, the total particle radius remains constant due to the formation of the ash layer, with an accompanying increase in particle porosity. Although the model can account for a particle size distribution of char, for the purpose of this study a uniform particle size has been considered. A mass balance is performed on selected individual particles throughout the bed, using a shrinking core modeling technique [3] to calculate the burning rate of the particles.

KILN COMBUSTOR MODEL

Previous mathematical models for rotary combustors have been developed and described by Essenhigh and Kuo [4], Gorog [5], Ghoshdastidar et al. [6], Jang and Acharya [7], Silcox et al. [8], Ghezzi et al. [9], Wormeck and Pershing [10], Lemieux et al. [11] and Sethi and Biswas [12].

In this study the kiln is divided into two active zones - a first zone for pyrolysis and a second zone for combustion. The model for the pyrolysis zone relies upon the following assumptions:

- bed height decreases linearly with distance along the kiln's centerline;
- the bed is well mixed in the vertical direction;
- only convective heat transfer from the gas to the bed is considered; the kiln wall is assumed to be refractory-lined, thus no heat is transferred through the barrel;
- the temperature in the product gas above the bed is constant throughout this zone of the kiln; its value is calculated by the method of Tillman [1];
- both the solid and gas regions operate in the plug-flow regime.

The model for the combustion zone is based upon the model for zirconium combustion, developed by Lemieux et al. [11] The bed depth and the uniform temperature throughout the bed are set at the values obtained at the exit of the pyrolysis zone. The model determines the burning rate of the char and calculates the char mass flow profile across the combustion zone. The bed is axially divided into slices of uniform surface oxygen concentration and each axial slice is divided vertically into segments of uniform oxygen concentration and particle size. A mass balance is performed on the slices in the vertical direction, yielding an ordinary differential equation describing diffusion of oxygen through a porous solid, with boundary conditions at the wall-solid and gas-solid interface. The particle burning rate (calculated as defined earlier) is combined with equations used to describe oxygen transport through the bed. Oxygen is assumed to be at a uniform concentration at each bed depth location, and the particles are assumed to be small enough that oxygen is at a uniform concentration in the gas surrounding each particle. The resulting finite difference equations are solved using a tridiagonal matrix algorithm.

The residence time of char in the kiln was calculated according to [13],

$$\theta = \frac{0.19L}{NDS}$$

where Θ is expressed in minutes, L is the kiln length, D, the kiln diameter, N, the number of revolutions of the kiln per minute and S is its inclination to the horizontal.

The complex three-dimensional nature of the kiln, with variations in the x, z and ϕ directions necessitates simplifications to enable a solution of the model equations. The bed height, t_b, is that corresponding to the one required for a bed with the same cross-sectional area and the same area exposed to the gas as in the actual kiln. It is reasonable to assume that the bed height remains constant in the burnout zone because char constitutes but a small fraction of the solids in this zone.

RESULTS FROM USING THE MODEL

Physical parameters required by the model were derived from the MSW incineration facility located at McKay Bay, a suburb of Tampa, Florida [14]. The sources of values of the various fundamental parameters required by the model are detailed in Reference 15. This reference also includes a discussion of the method used to solve the set of ordinary differential equations that constitute the pyrolysis model and the tridiagonal matrix, which is derived from the linear finite difference equations that constitute the combustion model. The FORTRAN code is also provided.

The cellulose and char mass flow profiles for a base case where the McKay Bay facility is operating at 77% of its design capacity are shown in Figures 1 and 2. Three distinct regions exist in Figure 1, which shows the results of the pyrolysis model. A significant portion of the kiln length is taken up by the heatup zone in which the MSW is raised to the reaction onset temperature. This is achieved by auxiliary fuel usage. Farther down the kiln is a short reaction zone in which pyrolysis of the waste occurs. This zone is characterized by a fall in the cellulose mass flowrate and a concomitant increase in that of the char. The third zone, where the pyrolysis model shows a constant char flowrate, and is in reality the burnout zone, is more appropriately described in Figure 2 by the results of the combustion model. Note that the abscissa has been shifted between Figures 1 and 2, such that the origin for Figure 2 is placed at the beginning of the burnout zone.

Bed temperature profiles through the heatup and reaction zones are shown in Figure 3 for the base case (9,400 kg waste/hr) and a case of further reduction in flow (4,500 kg waste/hr). This is an example of the use of the model to conduct parametric studies. Figure 3 indicates that the bed temperature is fairly insensitive to changes in the MSW feedrate. This could perhaps be justified by recalling that the bed is fairly well mixed and also that at a constant fuel-air ratio the flame temperature remains constant due to a balance between the reactive heat input and the convective heat removal. In operating incinerators the bed temperatures have been observed to be markedly different for different solids feedrates. This could result if the fuel-air ratio is not held constant during such experiments. Then again, incorporation of the radiative component of heat transfer could result in temperature profiles which match the actual trends better.

Another example of a parametric study is shown in Figure 4, which examines the relatively significant effect of bed porosity (0.54 void fraction - the base case - vs. 0.675 void fraction) on the rate of combustion in the burnout zone. Higher values of porosity reduce the length of this zone because the oxygen has better access to the char.

CONCLUSIONS

The mathematical model of a rotary kiln MSW combustor successfully predicts important aspects of its overall performance and several trends involving parametric variations in its operation. The size of the heatup zone, the size of the reaction zone, and the carbon burnout are quite accurate. However, temperature responses to changes in the MSW feedrate and the size of the burnout zone do not conform strictly to operational experience.

The current model, as well as improved future versions, should prove helpful in the analysis and design of waste-to-energy facilities employing this technology. The analysis of bed porosity has already shown that this parameter is a key factor in the performance of the burnout zone. Analysis of the heatup zone suggests that, because this region appears to require a major portion of the incinerator, preheat of the MSW feed by exiting flue gases might increase kiln capacity.

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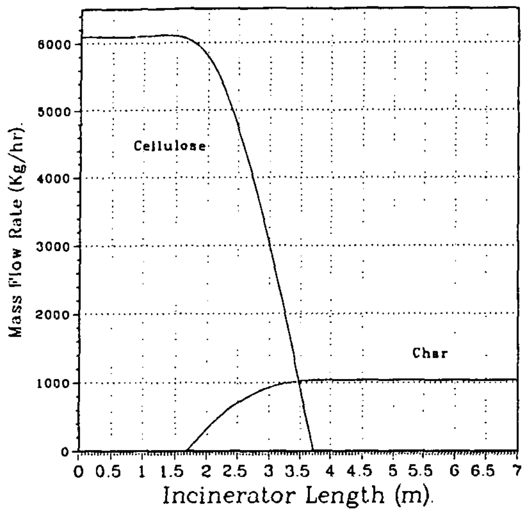

Figure 1. Mass Flow Profiles for the Base Case

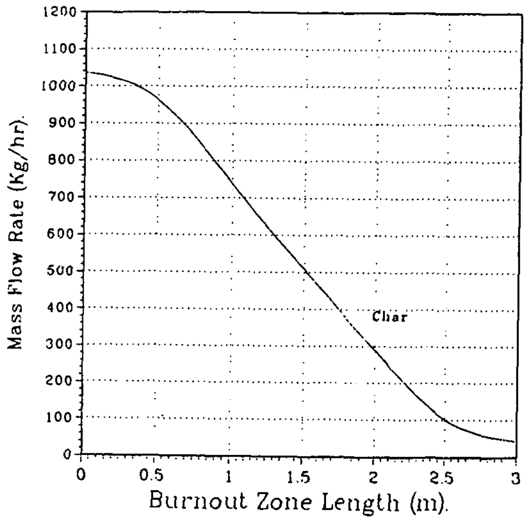

Figure 2. Char Mass Flow Profile in the Burnout Zone

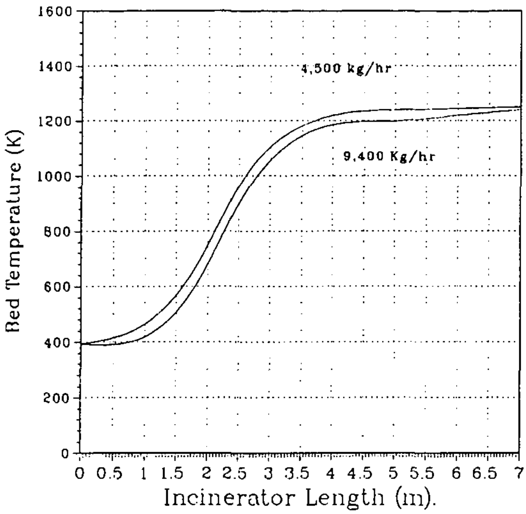

Figure 3. Bed Temperature Profiles

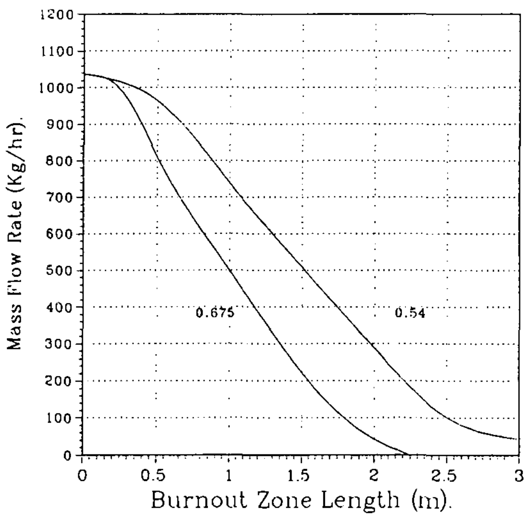

Figure 4. Effect of Bed Porosity