Droplet Motion under the Influence of Flow Nonuniformity and Relative Acceleration

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Abstract

A computational study on the dynamics of single droplets is performed in two gas flows at moderately higher Reynolds numbers, one is Poiseuille flow in which gas is either nitrogen or helium and the other one is counterflow formed by two opposed streams of nitrogen. The focus of the study is to review the methodologies used for representing the effects of flow nonuniformity and relative acceleration on droplet motion in moderately high Reynolds numbers. The motion of the droplets is observed to be affected by the flow nonuniformity and unsteadiness, characterized respectively by dimensionless parameters κ and A_C, and the effects due to nonuiformity and rate of change of relative velocity are separable. It is determined that acceleration and deceleration affect the drag and lift on droplets in dissimilar ways. The lift force caused by flow nonuniformity is in the same direction of κ in Poiseuille flow, whereas it is in the opposite direction of κ in counterflow. It is noted that the radius of curvature of droplet trajectory affects lift force more strongly than drag force. Modified correlations for the drag and lift coefficients as function of the Reynolds number and dimensionless parameters characterizing the flow nonuniformity and unsteadiness are proposed.

Keywords: drag, lift, unsteady, nonuniformity

NOMENCLATURE

A_C	Acceleration factor	m	Mass
C_A	Added-mass drag coefficient	Re	Droplet Reynolds number, $\mathrm{Re} = D_d V_r / \nu_g$
C_D	Drag coefficient	u_i	Velocity component in i-direction
C_{Ds}	Steady-state drag coefficient	V_r	Magnitude of relative velocity
C_H	Basset history drag coefficient	x_i	Displacement in i-direction
C_L	Lift coefficient	ρ	Density
D_d	Droplet diameter	μ	Viscosity
d_{ij}	The deformation rate tensor	ν	Kinematic viscosity
g	Gravity	κ	Nonuniformity factor
K	The coefficient of Saffman 's lift force		

Subscripts

g	Gas	i=1	Radial direction
d	Droplet	i=2	Axial direction

1. Introduction

One aspect of spray computation research which remains mostly unexplored is the accurate representation of the drag and lift forces operative on droplets as they undergo a highly complex, curvilinear, unsteady motion on a turbulent flow field. The droplet dynamics models being used currently in spray computations consider the standard drag force only; the effects of flow nonuniformity and droplet relative acceleration on the droplet drag and lift forces are not considered. In addition, the effect of unsteadiness on the motion of a droplet traveling in a curvilinear trajectory is not considered. A number of studies (Clift et al., 1978, Leal, 1980, and Puri and Libby, 1989, 1990) have found that these effects can significantly alter the droplet motion by changing the net drag force and introducing a significant lift force. Clearly, the trajectories obtained without consideration of these forces can be a significant source of error in a comprehensive spray computation.

In the analysis of multiphase flows, the particle shape is often assumed to be spherical for simplicity and the drag on a sphere is thought to have been well-understood at low Reynolds numbers. Many researcher have sought a general equation of motion to determine the trajectory of droplets in an unsteady, nonuniform flow. Originally Basset (1888), Boussinesq (1885), and Oseen (1927) developed a force expression, known as BBO equation, for a slowly moving, accelerating, rigid sphere in a still fluid. Later, Tchen (1947) extended the BBO equation to incorporate the effects of a temporally varying flow field on particle transport. Corrsin and Lumley (1956) modified Tchen's equation to account for spatial nonuniformity of the flow field. Riley (1971) revised Corrsin and Lumley's equation to properly account for the effect of the undisturbed flow on a particle's motion. Maxey and Riley (1983) modified the equation of Tchen (1947), and they suggested the following equation for a small rigid sphere in a nonuniform flow:

$$\frac{du_{di}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{C_{Ds}}{D_d}V_r\cdot(u_{gi}-u_{di}+\frac{1}{24}D_d^2\nabla^2u_{gi})+\frac{\rho_g}{\rho_d}\frac{Du_{gi}}{Dt}+\frac{1}{2}\frac{\rho_g}{\rho_d}\frac{d}{dt}(u_{gi}-u_{di}+\frac{1}{40}D_d{}^2\nabla^2u_{gi})$$

$$+\frac{\rho_g}{\rho_d}\sqrt{\frac{81\nu_g}{\pi D_d{}^2}}\int_{t_0}^{t}\frac{1}{\sqrt{t-t'}}\frac{d}{dt'}(u_{gi}-u_{di}+\frac{D_d^2}{24}\nabla^2u_{gi})dt'+(1-\frac{\rho_g}{\rho_d})g_i \tag{1}$$

The derivative d/dt denotes a time derivative following the moving sphere, and the derivative D/Dt the time derivative following a fluid element. The terms on the right hand side correspond in turn to the effects of viscous Stokes drag, pressure gradient of the undisturbed flow, added mass, Basset history term, and buoyancy.

The modified BBO equation and the above equation have been widely used for the study of the motion of small droplets in a fluid (Lázaro and Lasheras, 1989, and Liang and Michaelides, 1992). It should be noted, however, that both the equations are restricted to the Stokesian flow or "creeping flow", since the convective terms are omitted in their derivation. Unfortunately, no theoretical expression is available for the force on droplet at higher Reynolds number, if the effects like rotation, flow nonuniformity, and unsteadiness are added to the problem. Thus, some experimental work has been done to study the effects of flow nonuniformity and droplet acceleration at higher Reynolds number separately.

Odar and Hamilton (1964) used an experimental study and obtained correlations for the effects of added mass term and Basset history term at Reynolds number values up to 62. They expressed the total drag force by the use of the empirical coefficients C_{Ds}, C_A and C_H:

$$m_d \frac{du_{di}}{dt} = C_{Ds} \cdot \frac{\pi}{4} D_d^{\,2} \cdot \frac{1}{2} \cdot \rho_g V_r (u_{gi} - u_{di}) + C_A \cdot \frac{\pi}{6} \cdot D_d^3 \, \rho_g \frac{d}{dt}(u_{gi} - u_{di}) +$$

$$C_H \cdot \frac{D_d^2}{4} \cdot \sqrt{\pi \rho_g \mu_g} \cdot \int_{t_0}^{t} \frac{\frac{d}{dt}(u_{gi} - u_{di})}{\sqrt{t - t'}} \cdot dt' \tag{2}$$

where C_{Ds}, C_A and C_H are, respectively, the steady-state, added-mass and history drag coefficients. C_{Ds} is defined later in Eq. (14). Based on their measurements, Odar (1966) suggested the empirical formulas for C_A and C_H by introducing a nondimensional acceleration parameter A_C.

Odar (1966) confirmed that the empirical formulas for C_A and C_H, derived for a simple harmonic motion, are valid for the free fall of a sphere in a viscous fluid. Hughes and Gililand (1952) and Hjelmfelt and Mockros (1967) also predicted that a sphere which falls freely experiences drag higher than that given by the Stokes coefficient as it accelerates to its terminal velocity for higher Reynolds number. Tsuji and Tanaka (1990) investigated the drag on a sphere in a periodically pulsating flow experimentally for Reynolds number in the range 8000 < Re < 16,000. Their results show that the drag increases in the accelerating flow and decreases in the decelerating flow. Odar (1968) provided data on the drag of a sphere along a circular path in the Reynolds number range from 6 to 185 , which shows that the effects of the added mass and the history of the motion increase for this case whereas the contribution from the steady-state drag remains the same as that in a rectilinear motion. Contrary to the above, there is another group of works showing the opposite results. For instance, Temkin and Kim (1980) and Temkin and Melta (1982) obtained the drag by observing the motion of sphere in a shock tube and modified the drag coefficient C_D including the effects of unsteadiness. Their results show that acceleration decreases and deceleration increases droplet drag. Besides Temkin and Kim (1980) and Temkin and Mehta (1982), Ingebo (1956) reported results showing the same trend.

A thorough review of the effects of flow nonuniformity on particle motion is given by clift (1978) and leal (1980). The additional force caused by flow nonuniformity may be decomposed into a drag force in the direction of relative velocity and a lift force normal to the drag. In order to develop useful correlations, the effect of flow nonuniformity is usefully represented in terms of a nondimensional parameter κ and the droplet Reynolds number (Puri and Libby, 1990). Eichhorn and Small (1964) suspend large spheres in a Poiseuille flow at several inclinations of the tube and obtain lift and drag data in the Reynolds number range of 80 to 250. Saffman (1965) studies theoretically the lift on a small sphere in a slow shear flow. Dandy and Dwyer (1988) present numerical simulation for a neutrally buoyant spherical particle in a steady, linear shear flow over a Reynolds number range of ten to one hundred. Their results indicate that for a given rate of shear, the lift coefficient is inversely proportional to the square root of the Reynolds number for lower Reynolds number (less than ten) and constant at higher Reynolds number. Puri and Libby (1990) conduct experiments on droplets moving in a Poiseuille flow in the Reynolds number range of 0.7 to 27 and κ in the range of 10^{-3} to 6×10^{-2} and determined that the droplets experience drag larger than that indicated by the standard drag. Following the reasoning of Drew (1978) they attribute this increase in the drag to the effects of flow nonuniformity and empirically correlate the increase in drag and lift coefficients.

In spite of the abundance of literature on the effects of flow unsteadiness and nonuniformity, there are no previous correlation to calculate the drag and lift forces affected simultaneously by both flow nonuniformity and relative acceleration at higher Reynolds number. In the present paper, a computational study of motion of droplets in Poiseuille flow and counterflow is reported. The major focus of the study is to present a detailed comparison of the droplet trajectories predicted by five different approaches with the experimental data of Puri and Libby (1990, 1989) and to propose modified correlations for the effects of flow nonuniformity and relative acceleration at moderately high Reynolds number.

2. The Physical Situation

The droplet motions in Poiseuille flow and counterflow are studied. The flowfields are identical to those of Puri and Libby (1990) and Puri and Libby (1989), and the reader is refereed to their study for a detailed description. A Poiseuille flow is established in either nitrogen or helium at room temperature in a vertically mounted quartz tube of length 1.83 m and inner diameter of 2R=2.14 cm. Liquid droplets in an upward flowing Poiseuille flow of gases experience a downward velocity relative to the flow. A counterflowing flowfield is established by flowing gaseous nitrogen from two opposed ducts. The ducts have a radius of 2.3 cm and are placed 1.5 cm apart. The flow exits each duct with a discharge velocity of 31.7 cm/s. A droplet generator, the nozzle of which is placed in the bottom duct, introduces n-heptance droplets of 100 and 130 μm diameter into the gas stream just before it enters the counterflow. The flowfield is described by Libby et.al. (1989). The accuracy of the gas velocity components is confirmed by comparison with the experimental results of Chen et al. (1987) and the LDV measurements of Puri and Libby (1989).

3. The Equation of Motion

As reviewed above, several different approaches have been used in the past to represent the effects of acceleration and flow nonuniformity on droplet motion. The following approaches are employed in this paper.

Approach (1): The equation of motion, based on Eq.(1), in which the unsteady effect is introduced by using the empirical coefficients, C_A and C_H, and the lift force is included, is given as

$$\frac{du_{di}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{C_{Ds}}{D_d}V_r\cdot(u_{gi}-u_{di})+\frac{\rho_g}{\rho_d}\frac{Du_{gi}}{Dt}+C_A\frac{1}{2}\frac{\rho_g}{\rho_d}\frac{d}{dt}(u_{gi}-u_{di})$$

$$+C_H\frac{\rho_g}{\rho_d}\sqrt{\frac{81\nu_g}{\pi D_d^2}}\int_{t_0}^{t}\frac{1}{\sqrt{t-t'}}\frac{d}{dt'}(u_{gi}-u_{di})dt'+\frac{\rho_g}{\rho_d}\cdot\frac{2K\nu_g^{1/2}d_{ij}}{D_d(d_{lk}d_{kl})^{1/4}}(u_{gj}-u_{dj})+(1-\frac{\rho_g}{\rho_d})g \tag{3}$$

$$\frac{dx_i}{dt} = u_{di} \tag{4}$$

K=2.594 is the coefficient of Saffman's lift force, and the deformation rate tensor d_{ij} is defined as

$$d_{ij} = \frac{1}{2}(u_{gij}+u_{gji}) \tag{5}$$

where $$u_{g_{ij}} = \frac{\partial u_{gi}}{\partial x_j} \tag{6}$$

The expression of lift force used in Eq.(3) is a generalization of the expression provided by Saffman (1965) for three-dimensional shear fields, which is restricted to small droplet Reynolds number. In addition, the droplet Reynolds number based on the relative droplet velocity must also be smaller than the square root of the droplet Reynolds number based on the shear field. The formulas suggested by Odar (1966) are used to calculate C_A and C_H.

Approach (2): Following the equations suggested by Temkin and Mehta (1982) and others, the effect of unsteadiness is considered by modifying the drag coefficient C_D. The effect of flow nonuniformity on drag and lift is, however, represented in a manner similar to approach (1).

$$\frac{du_{di}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{C_D}{D_d}V_r\cdot(u_{gi}-u_{di}) + \frac{\rho_g}{\rho_d}\frac{Du_{gi}}{Dt} + \frac{\rho_g}{\rho_d}\cdot\frac{2K\nu_g^{1/2}d_{ij}}{D_d(d_{lk}d_{kl})^{1/4}}(u_{gj} - u_{dj}) + (1 - \frac{\rho_g}{\rho_d})g \tag{7}$$

d_{ij} and u_{gij} are the same as defined above.

Approach (3): The effects of flow nonuniformity and unsteadiness are represented in terms of additional lift and drag coefficients. If we assume that both lift and drag forces influence the droplet, then the force on the droplets acceleration components in the radial and axial directions are:

$$\frac{du_{d1}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{V_r}{D_d}[-C_L(u_{g2}-u_{d2}) + C_D(u_{g1}-u_{d1})\ ] \tag{8}$$

$$\frac{du_{d2}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{V_r}{D_d}[C_L(u_{g1}-u_{d1}) + C_D(u_{g2}-u_{d2})\ ] \tag{9}$$

where C_L and C_D are the coefficients of lift and drag respectively.

Puri and Libby (1990) suggest the following correlation for drag and lift coefficients:

$$C_D = C_{Ds}(1 + 575(\frac{\kappa^2}{\mathrm{Re}})^{3/4}) \tag{10}$$

$$C_L = 20C_{Ds}(\frac{\kappa^2}{\mathrm{Re}})^{3/4} \tag{11}$$

Approach (4): The modified correlations proposed in the present study are:

$$C_D = C_{Ds}(1 + C_{KD}(\frac{\kappa^2}{\mathrm{Re}})^{3/4}) - C_{AD}\cdot A_C \tag{12}$$

$$C_L = C_{KL} \cdot C_{Ds}(\frac{\kappa^2}{\text{Re}})^{3/4} - C_{AL} \cdot A_C \tag{13}$$

where C_{KD}, C_{AD}, C_{KL} and C_{AL} are constant.

In Poiseuille flow: $C_{AD} = 0.42$, $C_{AL} = 5 \times 10^{-3}$, when $A_C < 0.0$

$C_{KD} = 575.0$, $C_{KL} = 50.0$

In counterflow: $C_{AD} = 0.52$, $C_{AL} = 0.15$, when $A_C < 0.0$

$C_{AD} = 0.2$, $C_{AL} = 0.15$, when $A_C > 0.0$

$C_{KD} = 725.0$, $C_{KL} = 400.0$

A_C is defined by Temkin and Kim (1980).

Approach (5): The fifth approach considers only the viscous and pressure drag represented by C_{Ds}. For low Reynolds number, C_{Ds} is given by the Stokes drag, whereas for high Reynolds number, it involves Stokes drag and a correction such as proposed by Putnam (1961), i.e.,

$$C_{Ds} = \frac{24}{\text{Re}} \cdot (1 + \frac{\text{Re}^{2/3}}{6}) \tag{14}$$

4. Results and Discussion

The fourth order Runge-Kutta method has been used to calculate the droplet velocity and displacement. The effects of flow nonuniformity and relative acceleration are investigated by studying the droplet trajectories and displacement histories in both radial and axial direction predicted by the five approaches and experimental data.

Figures 1 shows the droplet trajectories and displacement histories in both radial and axial directions predicted by above five approaches, and obtained experimentally in Poiseuille flow. Three cases have been considered and each case has different initial conditions, and also different droplet size or different fluid. In this paper only one case is shown. As seen in Fig.1, the droplets introduced off the axis migrate toward the axis. Comparing the displacement histories in radial direction, it is noted that the values predicted by approach (5) are greater than those determined experimentally. In addition, the existence of a lift force which moves the droplet towards the axis is indicated. The sign of the lift force is the same as that given by Saffman (1965). Compared with experimental data, the approach (3) underpredicts the lift force whereas approach (1) and (2) overpredict the lift force. Note that the error in the trajectory prediction is mainly due to the inaccurate representation of the lift force. In approach (1) and (2), the lift force is evaluated by using the Saffman lift force expression, which is restricted to low Reynolds number situations. Approach (3), based on the correlation of Puri and Libby (1990), considers the flow nonuniformity effect, but may be improved further by including the acceleration effect. The modified equation (13) used in approach (4) includes the latter effect, and provides a better representation for the lift coefficient.

The droplet displacement in axial direction is influenced mostly by drag force. The larger the drag force, the shorter the distance traveled by the droplet in the axial direction, when droplets move in the opposite direction of gas flow. As demonstrated in Fig.1, approach (1) and (2) underpredict the drag force. A plausible explanation for the underprediction of the drag force by approach (1) is that it employs correlations of Odar and Hamilton (1964), which are based on an experimental study of droplets in harmonic motion. If the droplet moves along a curved path, the unsteady effect will increase. As the result of curvilinear trajectory of droplets and inaccurate consideration of nonuniformity, the approach (1) underpredict the drag force. Similarly the error in using approach (2) is caused by an inappropriate application of the formula proposed by Temkin and Melta (1982) and Saffman (1965) lift force expression. In order to modify approach (3) which underpredicts the drag force, we include the unsteady effect. Consequently, the droplet trajectories as well as displacement histories in both radial and axial directions predicted by the modified correlation are in better agreement with those determined experimentally.

The droplets in counterflow experience a much more complex, curvilinear, and unsteady (including both acceleration and deceleration) motion. Two cases have been studied with different droplet size and initial condition. In case 1, droplet diameter is 100 μm. In case 2, droplet size is 130 μm, and the droplet initial velocity in axial direction is much higher than that in radial direction.

As noted from Figs.2 and 3, the trajectories predicted by approach (5) are much different from the experimental data, especially in radial direction, indicating lift force must be important in these cases. The presence of lift in a curvilinear trajectory is not surprising. In experiments on the motion of a sphere along a curvilinear path in the Reynolds number range of 30 to 80, Odar (1968) finds that the lift is as high as ten percent of drag. In their study on droplets in a counterflow, Puri and Libby (1989) contend that the droplets are influenced by the skewness of the acceleration vector from the relative velocity vector. As a result, the net force due to acceleration is not collinear with the relative velocity. Consequently, the consideration of acceleration effects in a curved trajectory requires that both the drag and lift due to acceleration must be calculated. The unsteady effect on drag and lift is given in Eq.(12) and Eq.(13). From these relations it is seen that the deceleration will increase drag force, and acceleration will decrease drag force. This is consistent with the results of Temkin and Mehta (1982).

It is known that the flow nonuniformity affects both drag and lift force. It is not clear, however, as to how the direction of lift force changes with the sign of κ. In Poiseuille flow, κ is always positive. According to Saffman, if the particle lags behind the fluid, a radially inward lift force exists causing their migration toward the tube axis. If, on the other hand, the particle travels faster than the fluid, the effect will move the particle away from the axis, i.e., the lift force coefficient follows the sign of κ. In counterflow, the plots of C_L and κ indicate that the direction of C_L is opposite to that of κ. In the present study, this observation is used to determine the sign of C_L in Eqs. (12) and (13).

For approach (4), comparing the constants C_{AD}, C_{AL}, C_{KD}, and C_{KL} in Poiseuille flow with those in counterflow, it is noted that the constants C_{AD} and C_{KD} used in calculating drag coefficient are not much different in the two flows. However, the constants C_{AL} and C_{KL} used in calculating lift coefficient are much larger in counterflow than those in Poiseuille flow. It indicates that the radius of curvature of droplet trajectory, which is much larger in Poiseuille flow than in counterflow, but change continuously along the droplet trajectory in the counterflow, may affect the lift force, and the larger the radius of the curvature of droplet trajectory, the less effect on the lift force. Finally, a plausible explanation for the departure of displacement histories in radial direction predicted by approach (4) and experimental data (Figs. 2(b) and 3(b)) is attributed to the fact that the correlations used in approach (4) do not consider the effect of changing curvature and skewness of acceleration vector along the droplet trajectory.

5. Conclusions

The droplet motion under the influence of flow nonuniformity and relative acceleration has been investigated in Poiseuille flow and counterflow. Several approaches that are currently in use for representing these effects have been evaluated. It is found that the application of Odar's formula, Temkin's formula and Puri and Libby's correlation is not accurate enough to predict the trajectories obtained from previous experimental studies. It is indicated that calculations of κ and A_C can be performed for both Poiseuille flow and counterflow, so that the effects due to nonuniformity and rate of change of relative velocity are separable. It is determined that acceleration and deceleration affect the drag on droplets in dissimilar ways, which is consistent with the results of Temkin and Mehta (1982). The lift force caused by flow nonuniformity is in the same direction of κ in Poiseuille flow, and in the opposite direction of κ in counterflow. It is seen that the radius of curvature of droplet trajectory affects lift force more strongly than drag force, and the larger the radius of the curvature of droplet trajectory, the less effect on the lift force. Modified correlations for the drag and lift coefficients as function of the Reynolds number and dimensionless parameters characterizing the flow nonuniformity and unsteadiness are proposed.

Since the correlations proposed in the present work is based on the analyses of the experimental data of Puri and Libby (1990, 1989), they may not be applicable to other situations that are significantly different from these experiments. The effects of the radius of curvature of droplet trajectory and the skewness of the acceleration vector from the velocity vector on the drag and lift force have not been studied in detail in the present work. Clearly, more experimental and theoretical studies are needed to analyze these effects on the drag and lift force.

Acknowledgments

This work has been supported by a grant from the NASA Lewis Research Center under the technical direction of D. Bulzan, Institute of Computational Mechanics in Propulsion (ICOMP). We thank Dr. I. S. Puri for helpful discussions.

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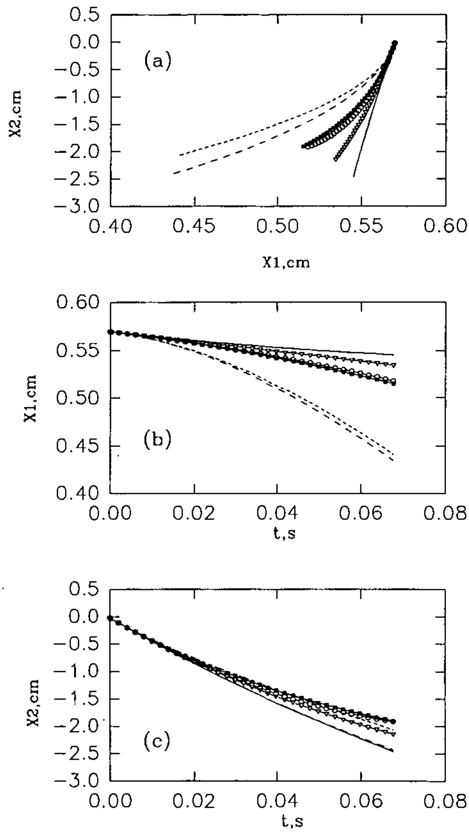

FIGURE 1 Droplets in Poiseuille flow : case 1
(a) Trajectories.
(b) Displacement histories in radial direction.
(c) Displacement histories in axial direction.

- - - Approach 1	------ Approach 2	▾ Approach 3
• Approach 4	—— Approach 5	∘ Exp. data

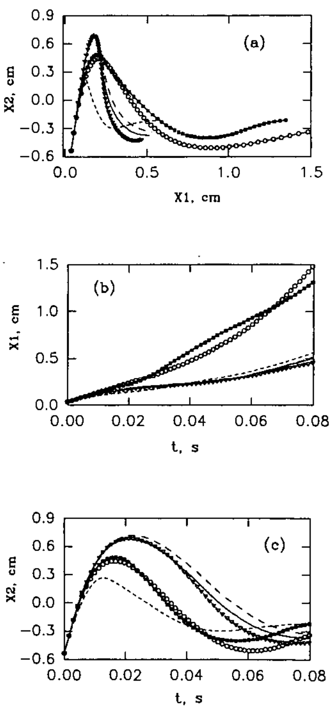

FIGURE 2 Droplets in counterflow : Dd=100E−6 (m)

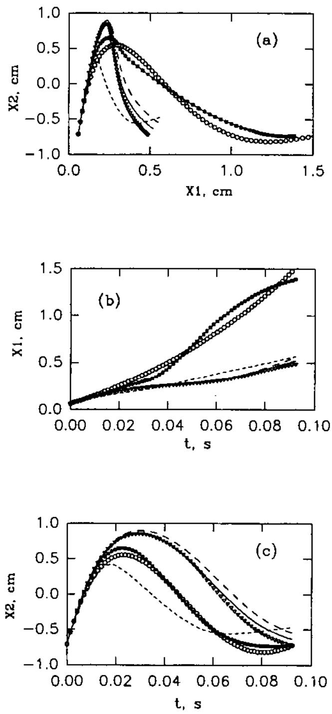

FIGURE 3 Droplets in counterflow : Dd=130E−6 (m)

(a) Trajectories.
(b) Displacement histories in radial direction.
(c) Displacement histories in axis direction.

- - - Approach 1	------ Approach 2	• Approach 3
▪ Approach 4	—— Approach 5	∘ Exp. data