Optical Integral Differences:
Kinetic Energy Savings or a Cut-off Effect?

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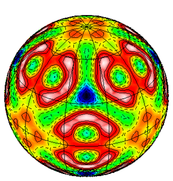
Norman and Chubukov, Phys Rev B 73, 140501 (2006)



Rome - July 2, 2007

Optical Integrals

$\int d\omega \, \text{Re} \, \sigma(\omega) = \omega_{pl}^2/8$

Sum all bands -> proportional to n/m (f sum rule)

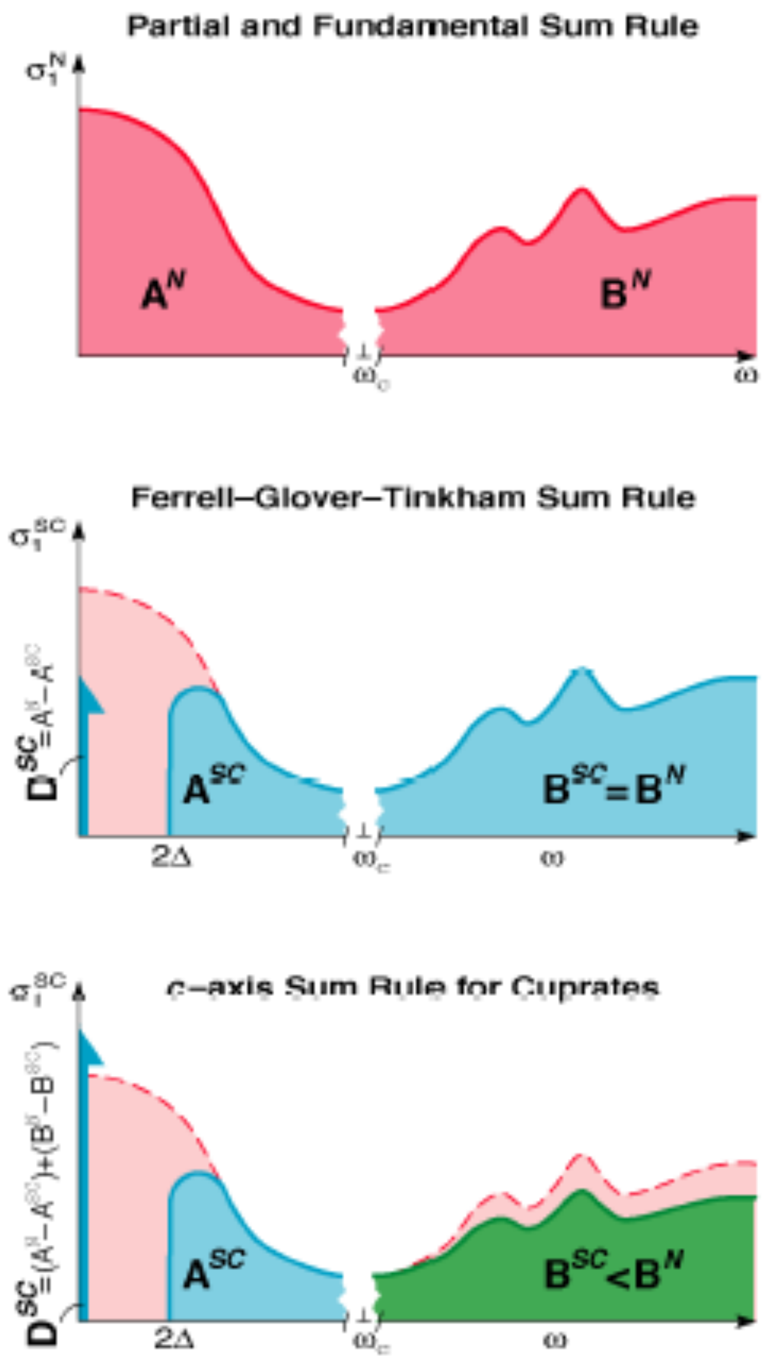
Single band only -> proportional to E_K

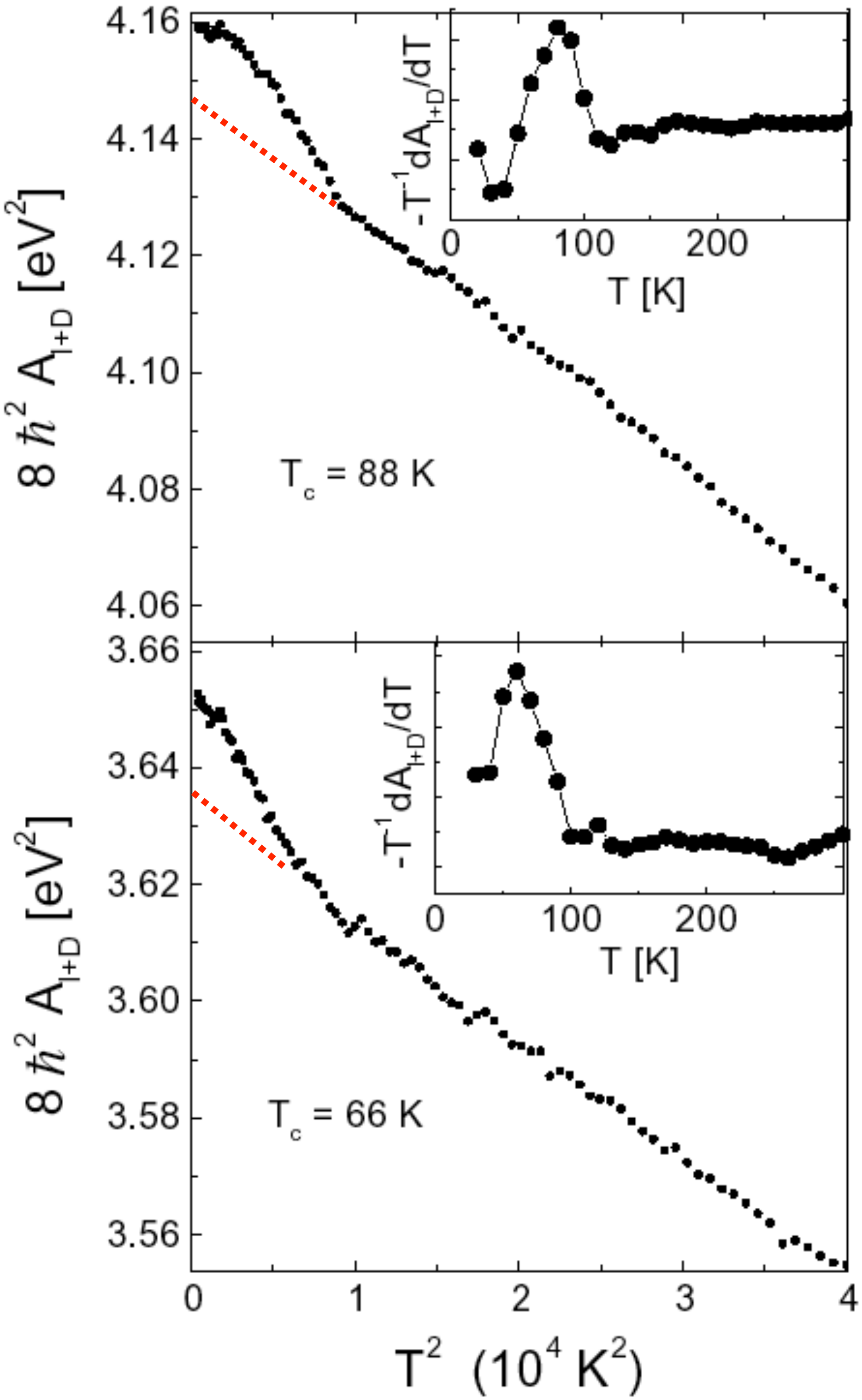
$E_K = \Sigma_k \, (\partial^2 \varepsilon_k / \partial k^2) \, n_k$

$E_{kinetic} = \Sigma_k \, \varepsilon_k \, n_k$

In general, E_K is *not* a conserved quantity, though it is for a parabolic dispersion - $k^2/2m^*$

Sum Rule - Klein and Blumberg, Science 283, 42 (1999)



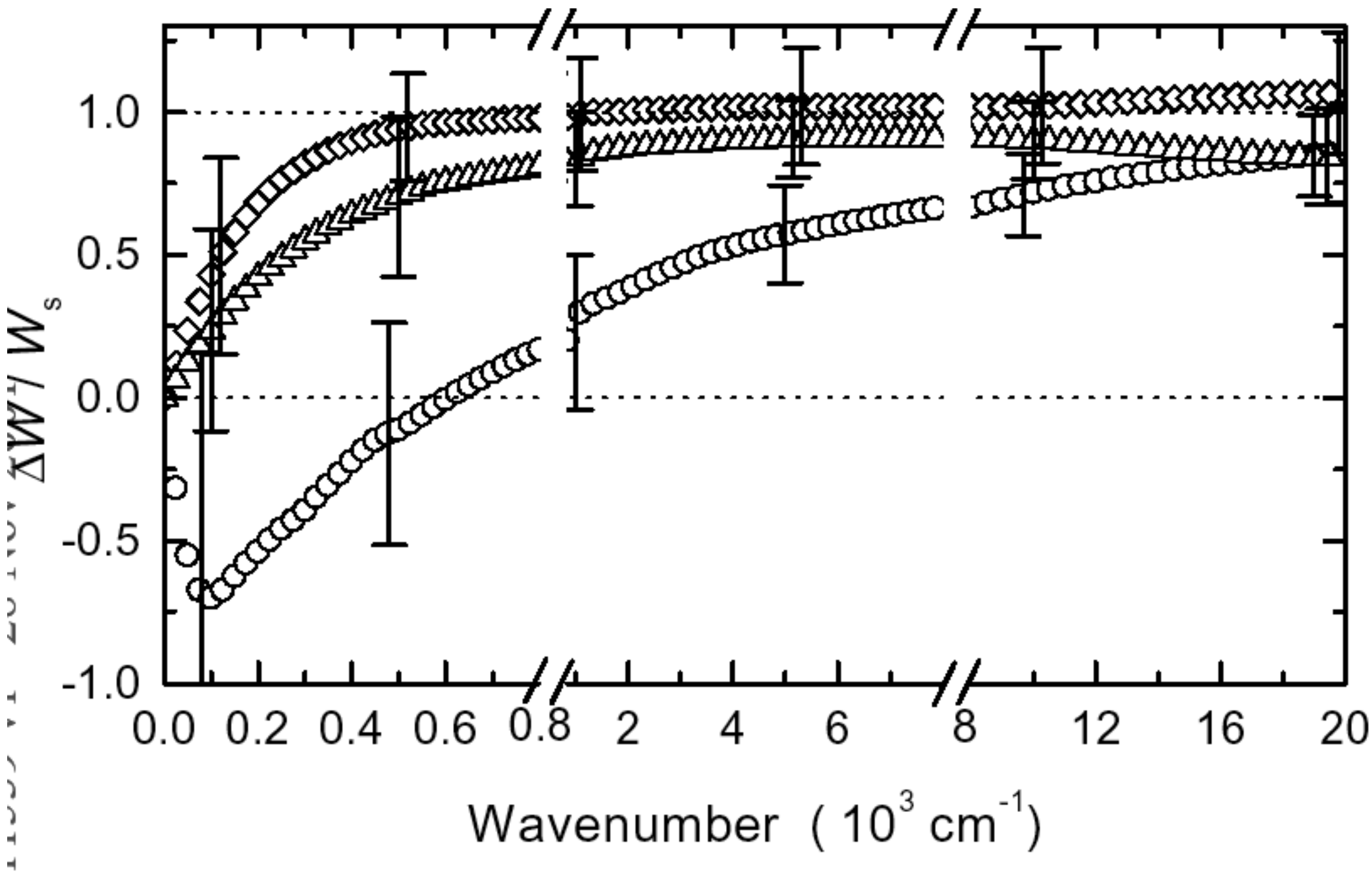


An increase is observed in the optical weight below T_c relative to the extrapolated behavior from above T_c

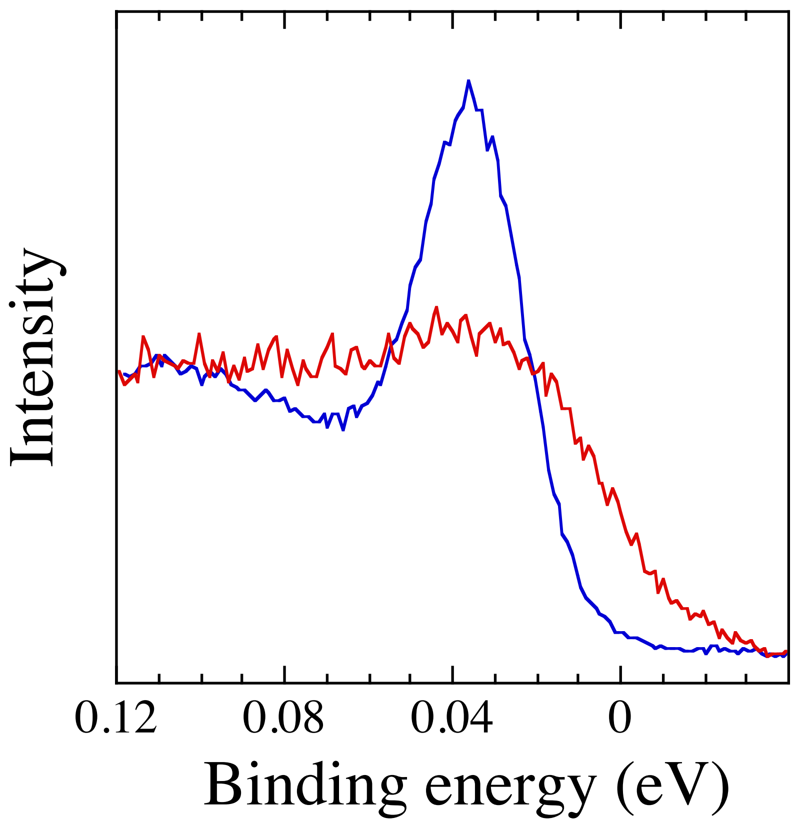
Molegraaf & van der Marel
Science 295, 2239 (2002)

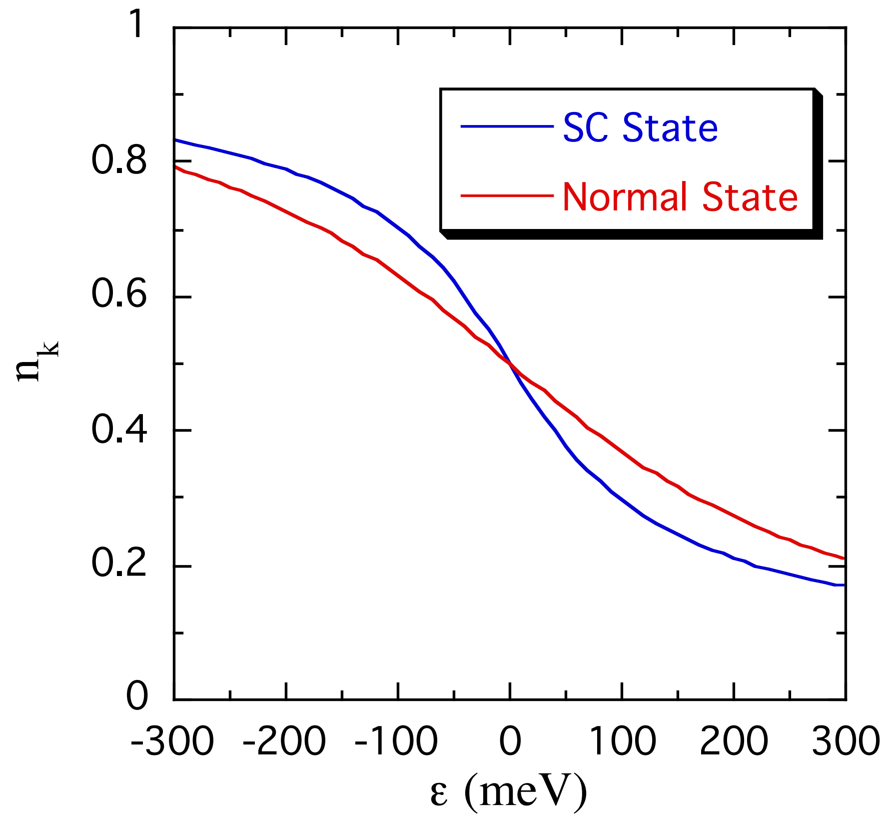
A similar effect has been seen in Bi2212 by the ESPCI group

Santander-Syro, Lobo, Bontemps, Europhysics Letters 62, 568 (2003)



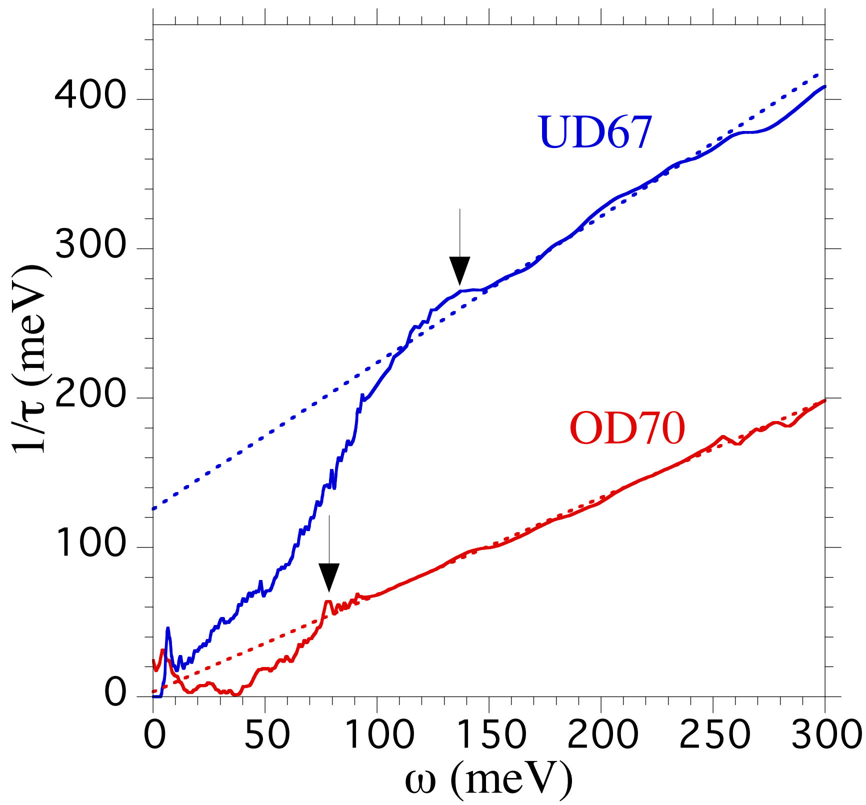
The onset of coherence below T_c as observed in ARPES would imply that the resulting gain in kinetic energy could exceed the loss of kinetic energy due to pair formation

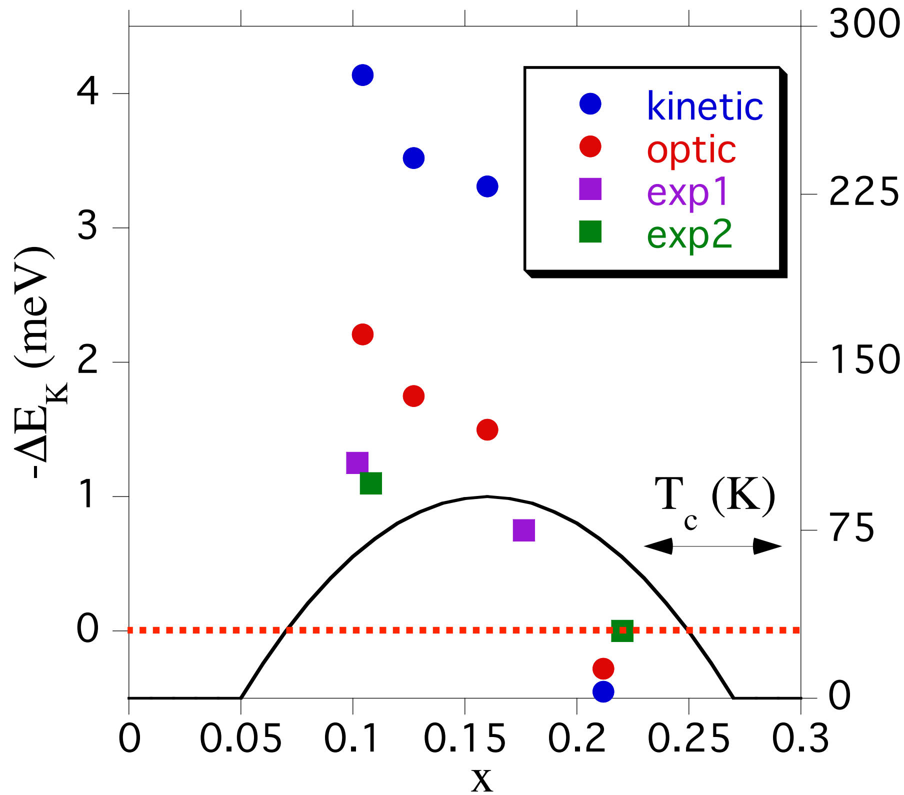




Norman, Randeria, Janko, Campuzano, Phys Rev B 61, 14742 (2000)

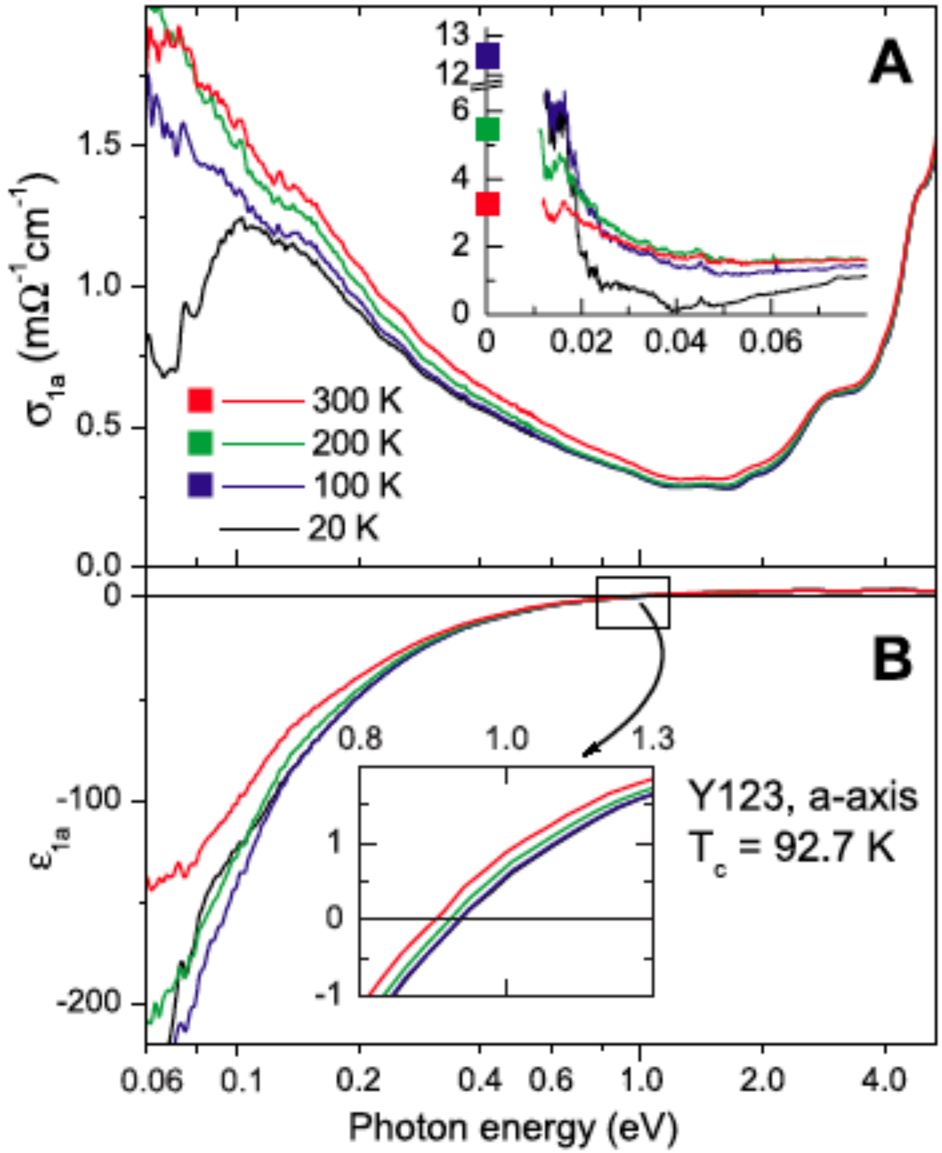
Detailed calculations of the optical integral and the kinetic energy based on a self-energy with a scattering rate gap below T_c confirm this picture

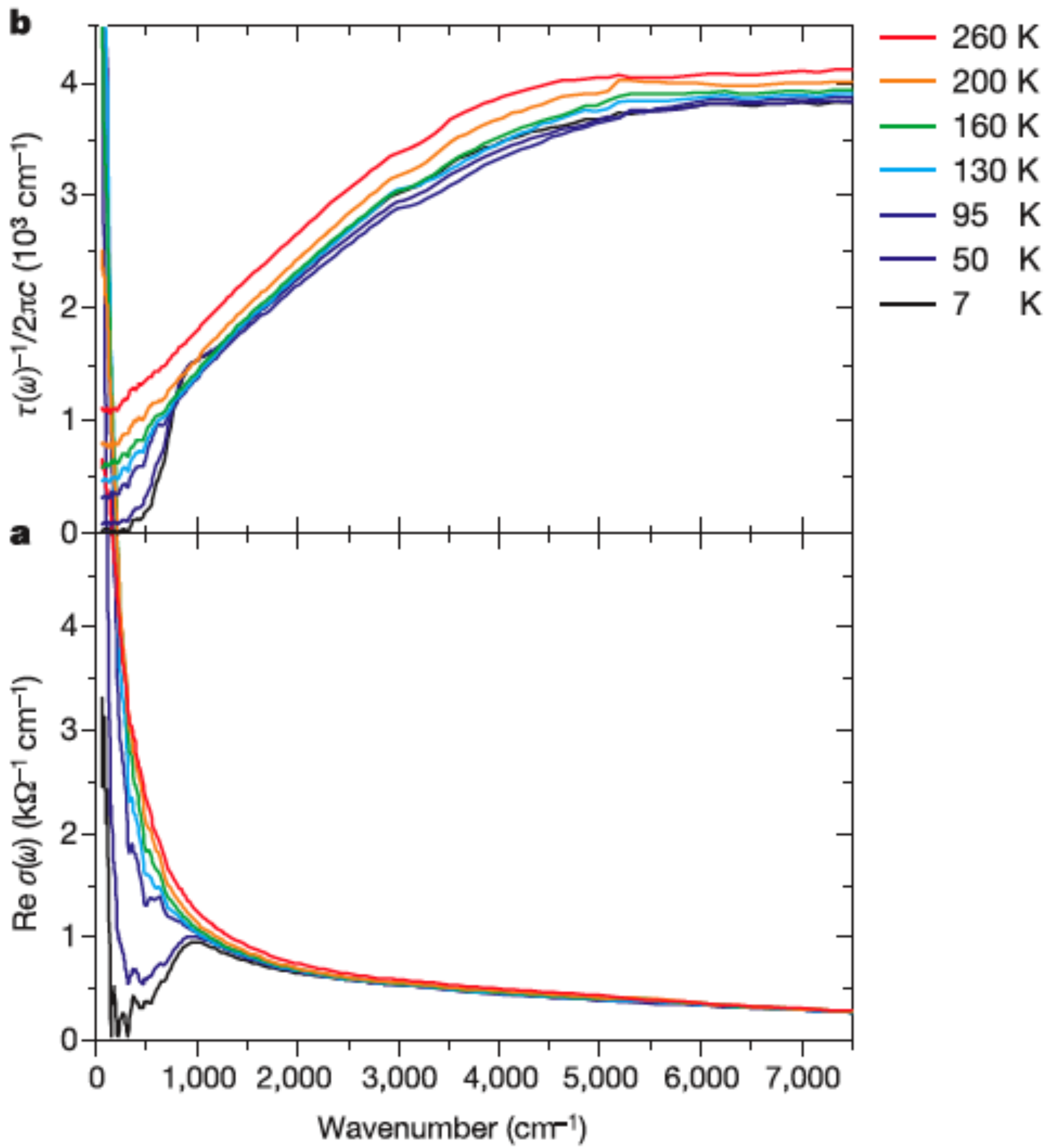




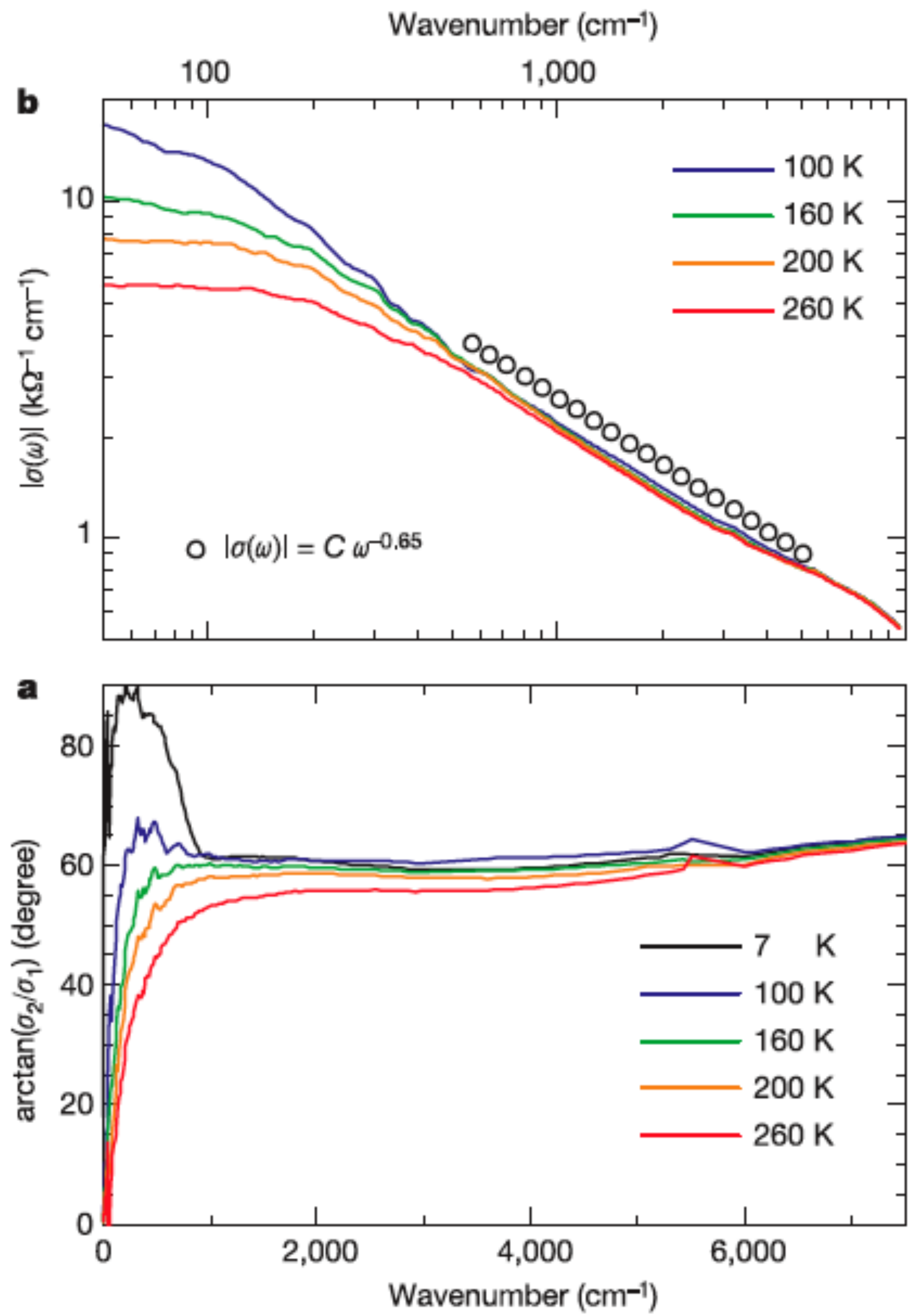
Norman and Pepin, Phys Rev B 66, 100506 (2002)

But Boris *et al.* - Science 304, 708 (2004) - claim spectral weight <u>loss</u> below T_c (that is, kinetic energy loss)





Van der Marel *et al.*, Nature 425, 271 (2003)


Wavenumber (cm^{-1})
100
1,000
b
10
1
$|\sigma(\omega)|$ (kΩ^{-1} cm^{-1})
100 K
160 K
200 K
260 K
$|\sigma(\omega)| = C\,\omega^{-0.65}$
a
80
60
40
20
0
arctan(σ_2/σ_1) (degree)
7 K
100 K
160 K
200 K
260 K
0
2,000
4,000
6,000
Wavenumber (cm^{-1})

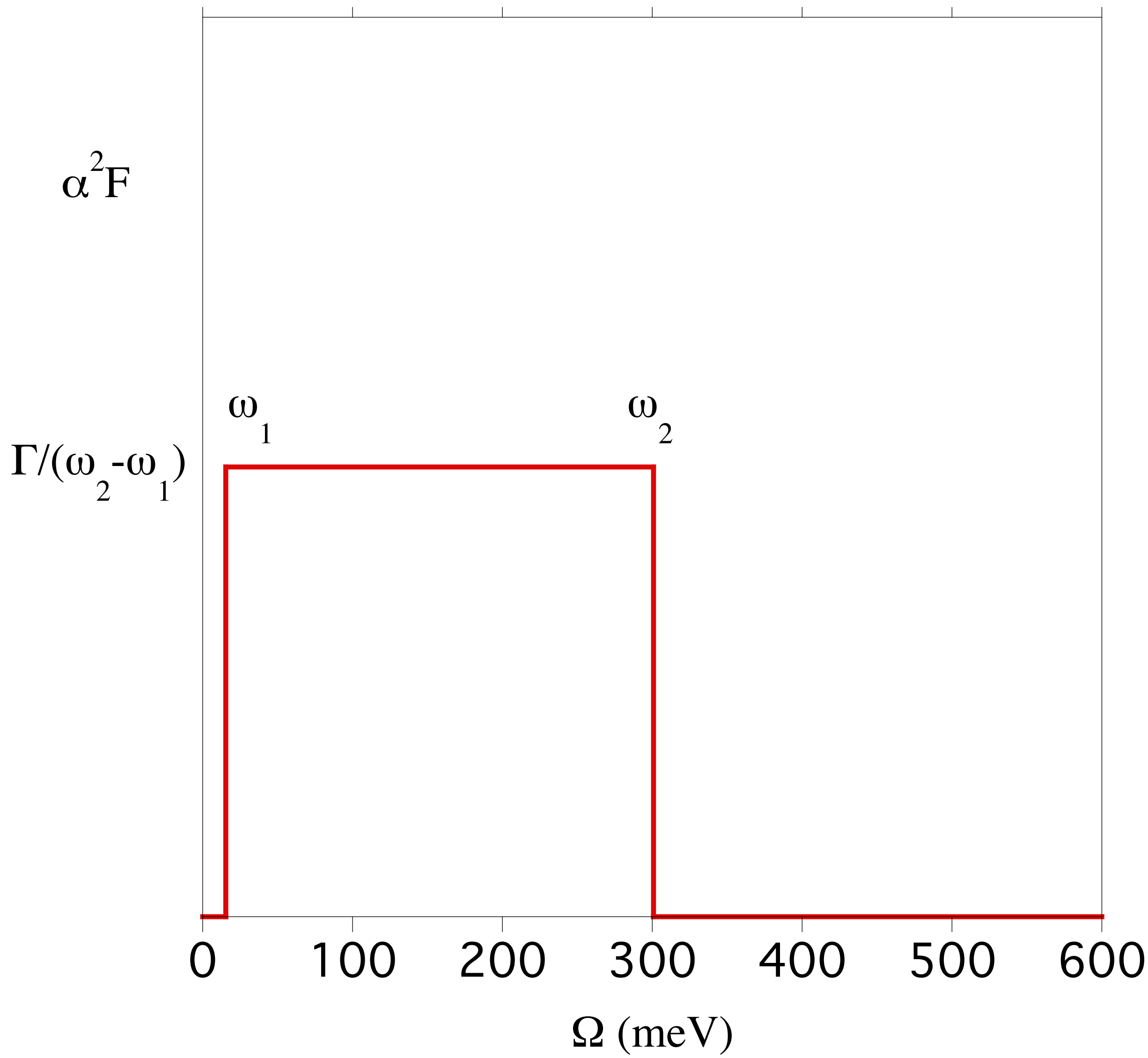
$$\sigma(\omega) = \frac{\omega_{pl}^2}{4\pi} \int d\epsilon \frac{f(\epsilon) - f(\omega + \epsilon)}{i\omega} \frac{1}{-\omega - \Sigma^*(\epsilon) + \Sigma(\omega + \epsilon)}$$

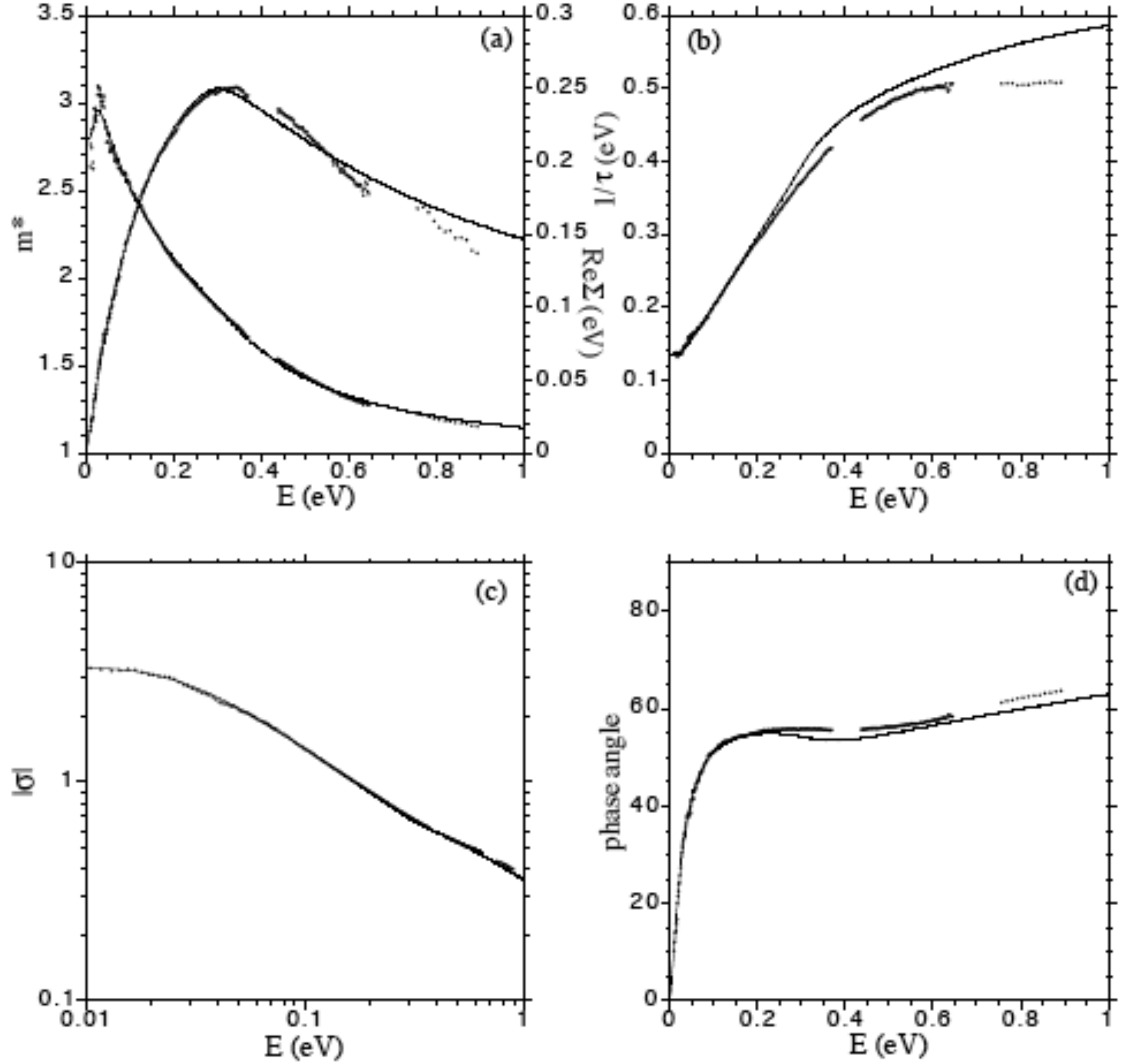
$$\Sigma(\omega) = \int \frac{d\Omega}{\pi} \int d\epsilon \alpha^2 F(\Omega) \frac{n_B(\Omega) + f(\epsilon)}{\omega - \epsilon + \Omega + i\delta}$$

$$\sigma(\omega) = \frac{\omega_{pl}^2}{4\pi} \frac{1}{1/\tau(\omega) - i\omega m^*(\omega)}$$

$$1/\tau(\omega) = \frac{1}{\omega} \int_0^\infty d\Omega \alpha^2 F(\Omega)[2\omega \coth(\frac{\Omega}{2T}) \\ -(\omega + \Omega) \coth(\frac{\omega + \Omega}{2T}) + (\omega - \Omega) \coth(\frac{\omega - \Omega}{2T})]$$

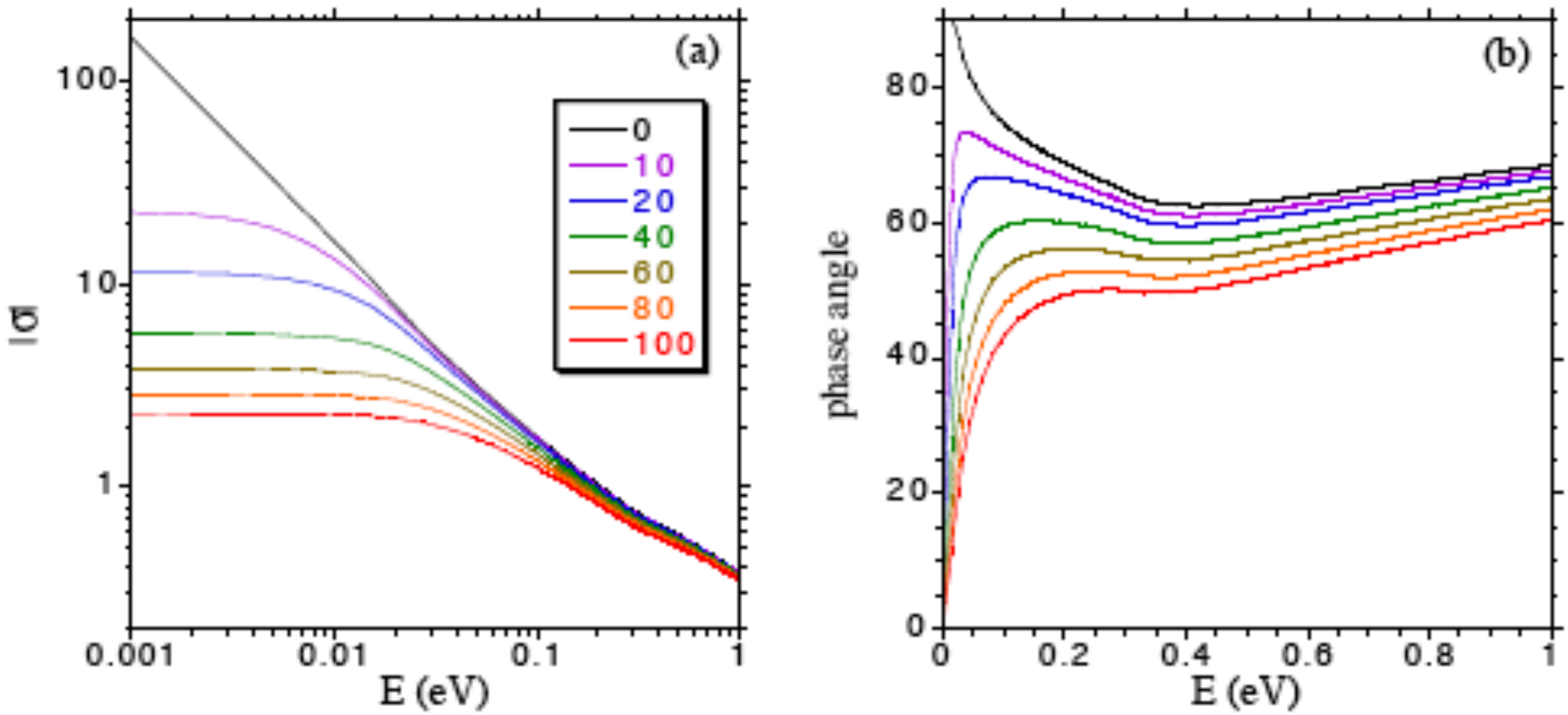
Example - "Gapped Marginal Fermi Liquid"



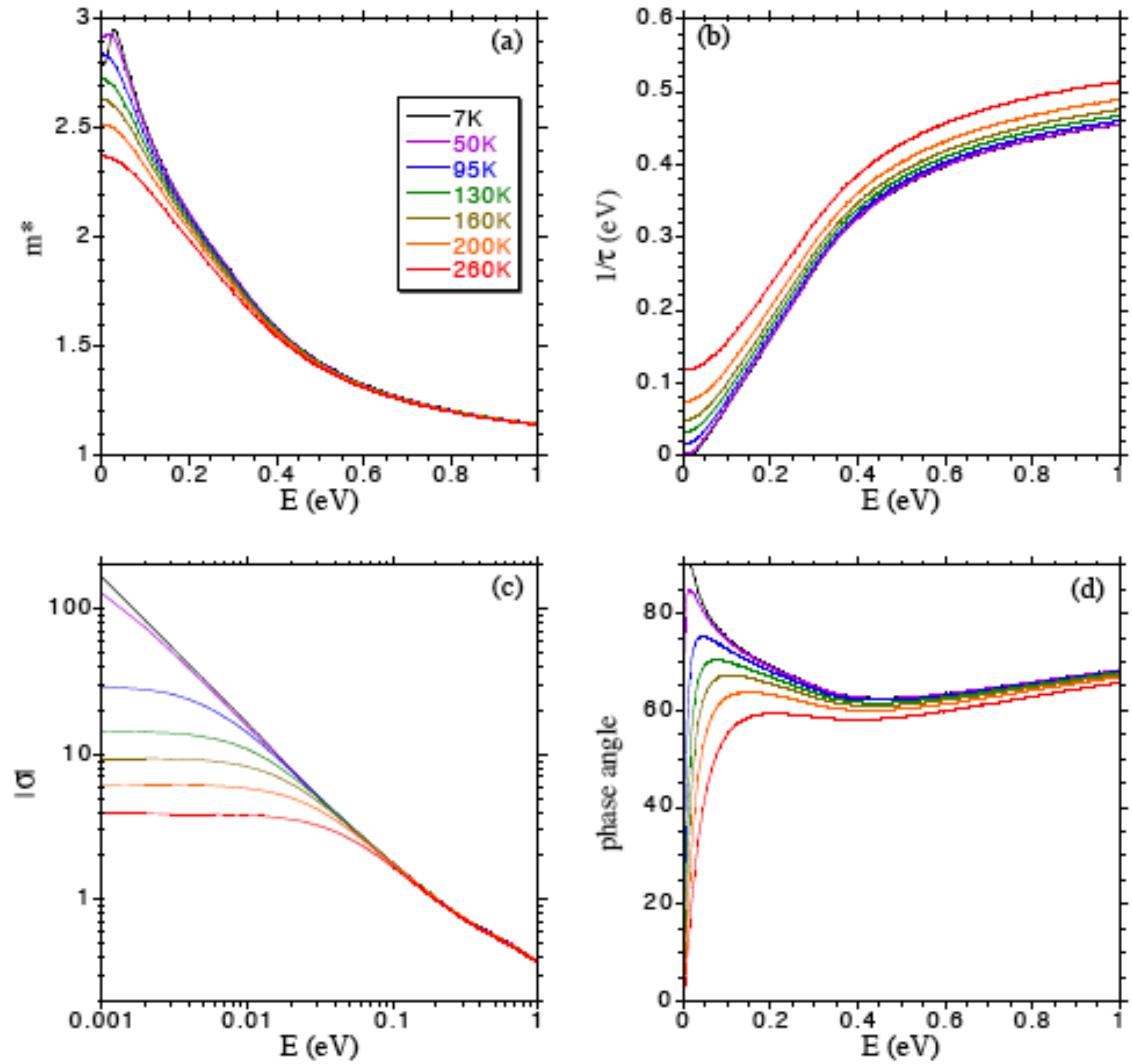


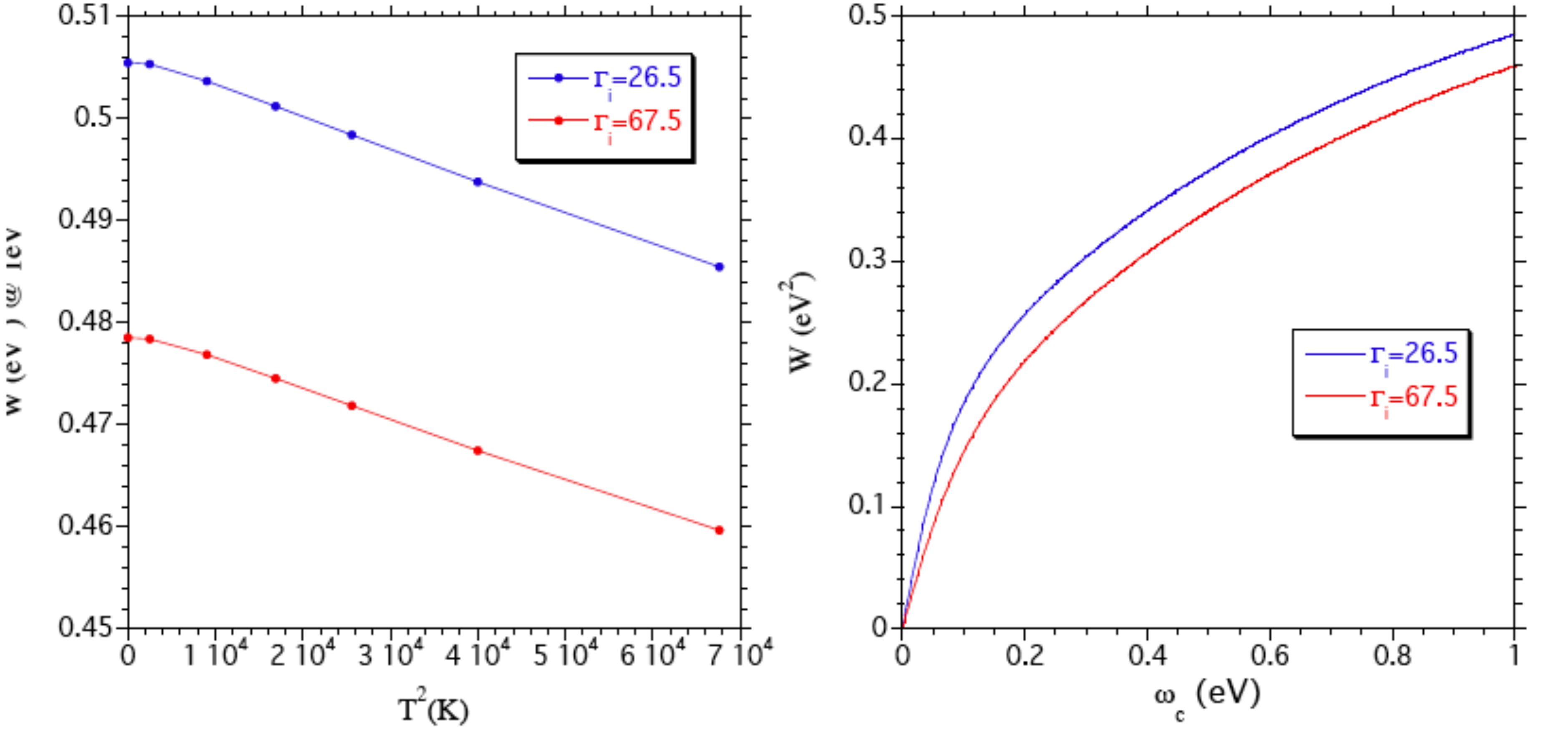
(a)
m*
ReΣ (eV)
E (eV)
(b)
1/τ (eV)
E (eV)
(c)
|σ|
E (eV)
(d)
phase angle
E (eV)

As a function of impurity scattering rate

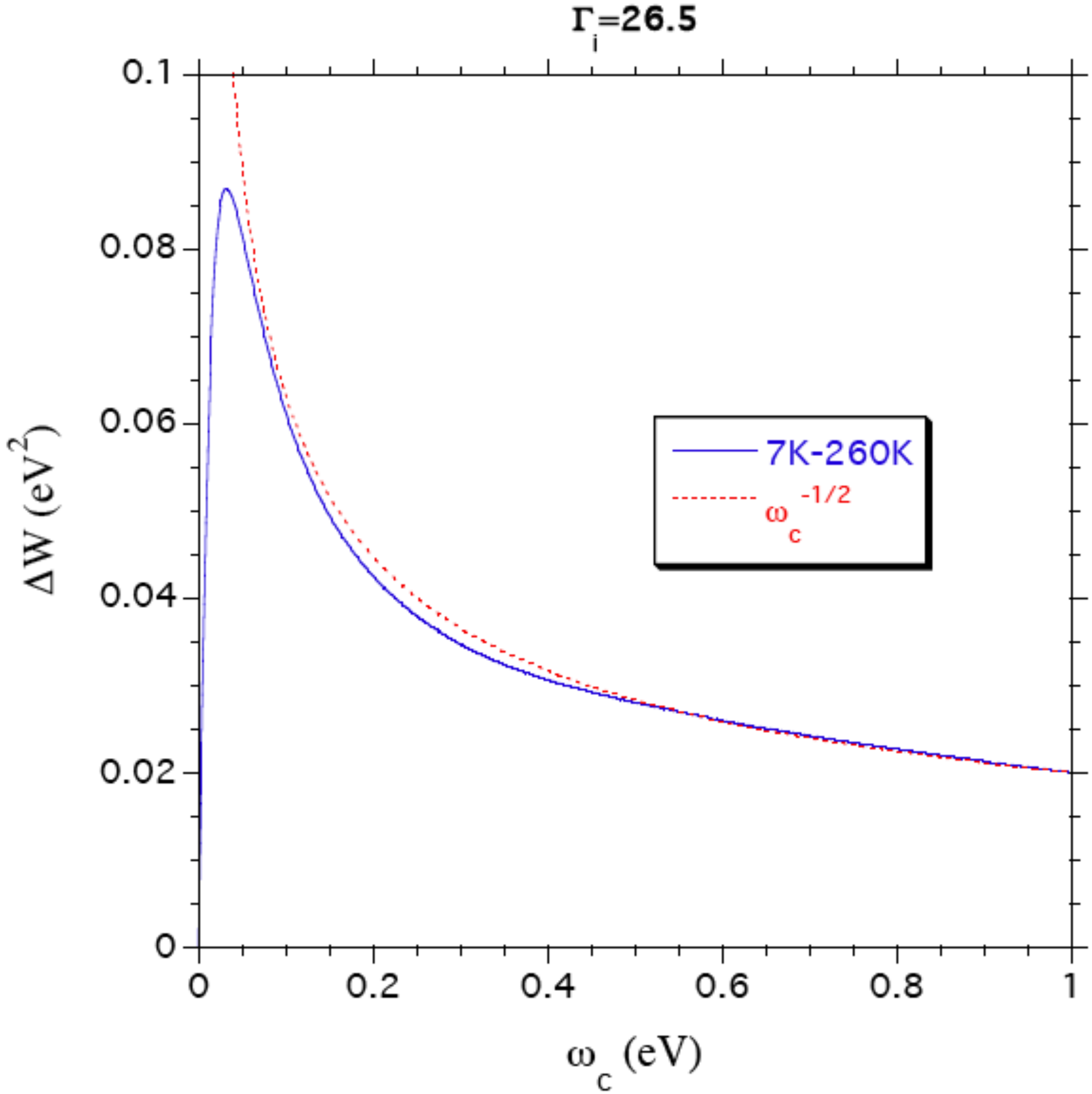


As a function of temperature

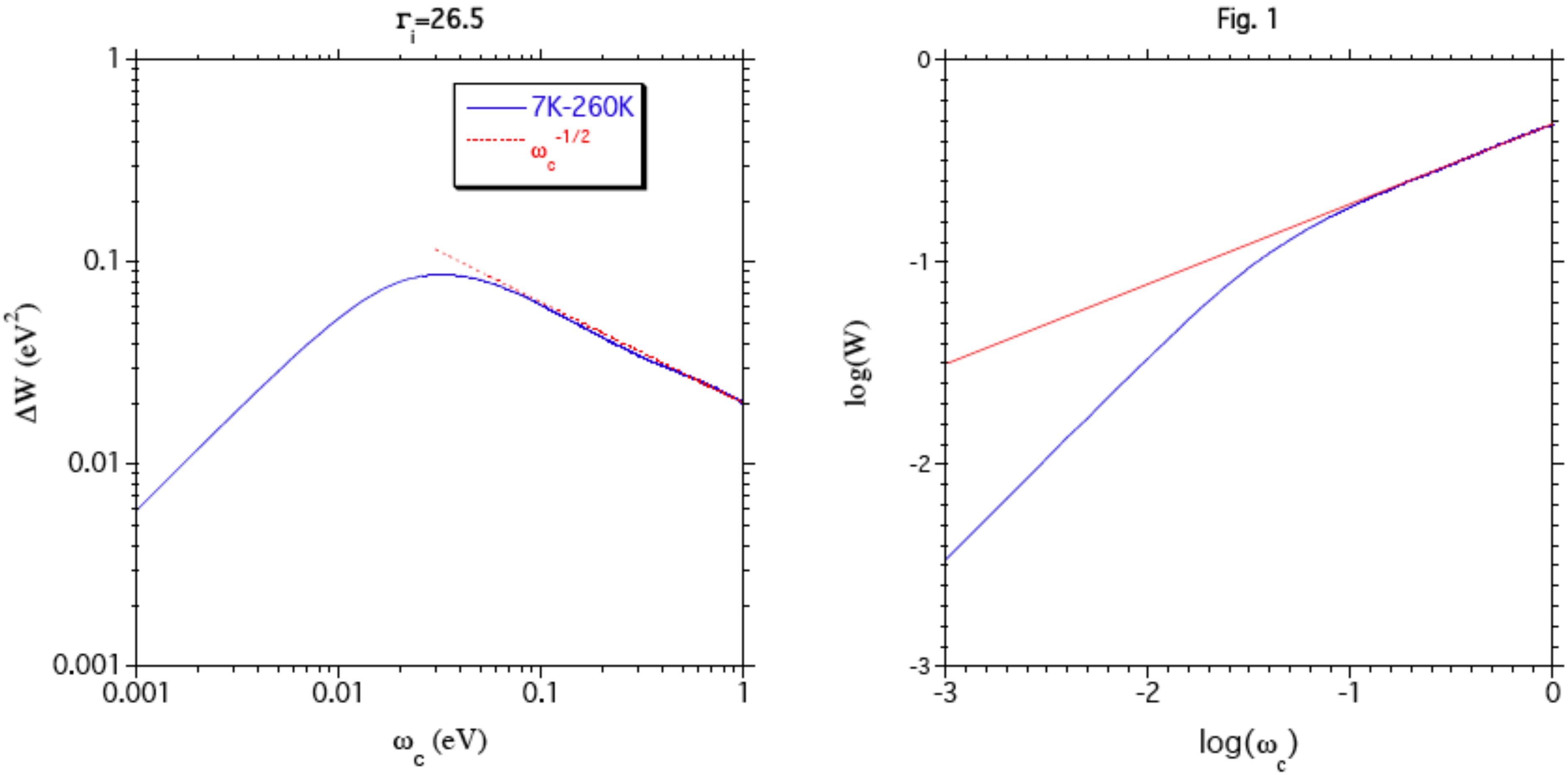




Γi=26.5
Γi=67.5
0.51
0.5
0.49
0.48
0.47
0.46
0.45
0
1 10^4
2 10^4
3 10^4
4 10^4
5 10^4
6 10^4
7 10^4
T^2(K)
W (eV^2)
Γi=26.5
Γi=67.5
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
ωc (eV)

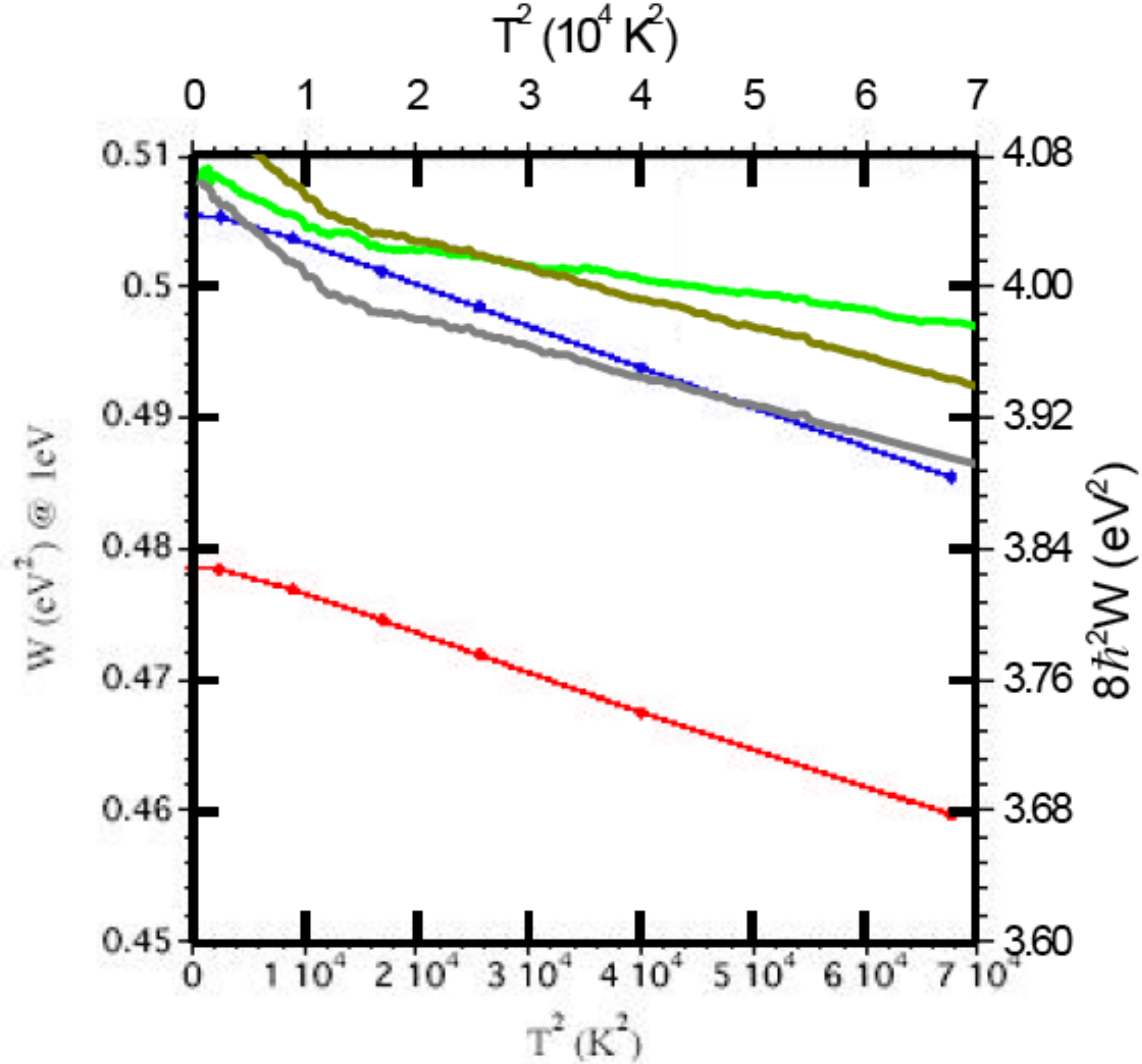


Γi=26.5
ΔW (eV2)
ωc (eV)
7K-260K
ωc-1/2
0
0.02
0.04
0.06
0.08
0.1
0
0.2
0.4
0.6
0.8
1

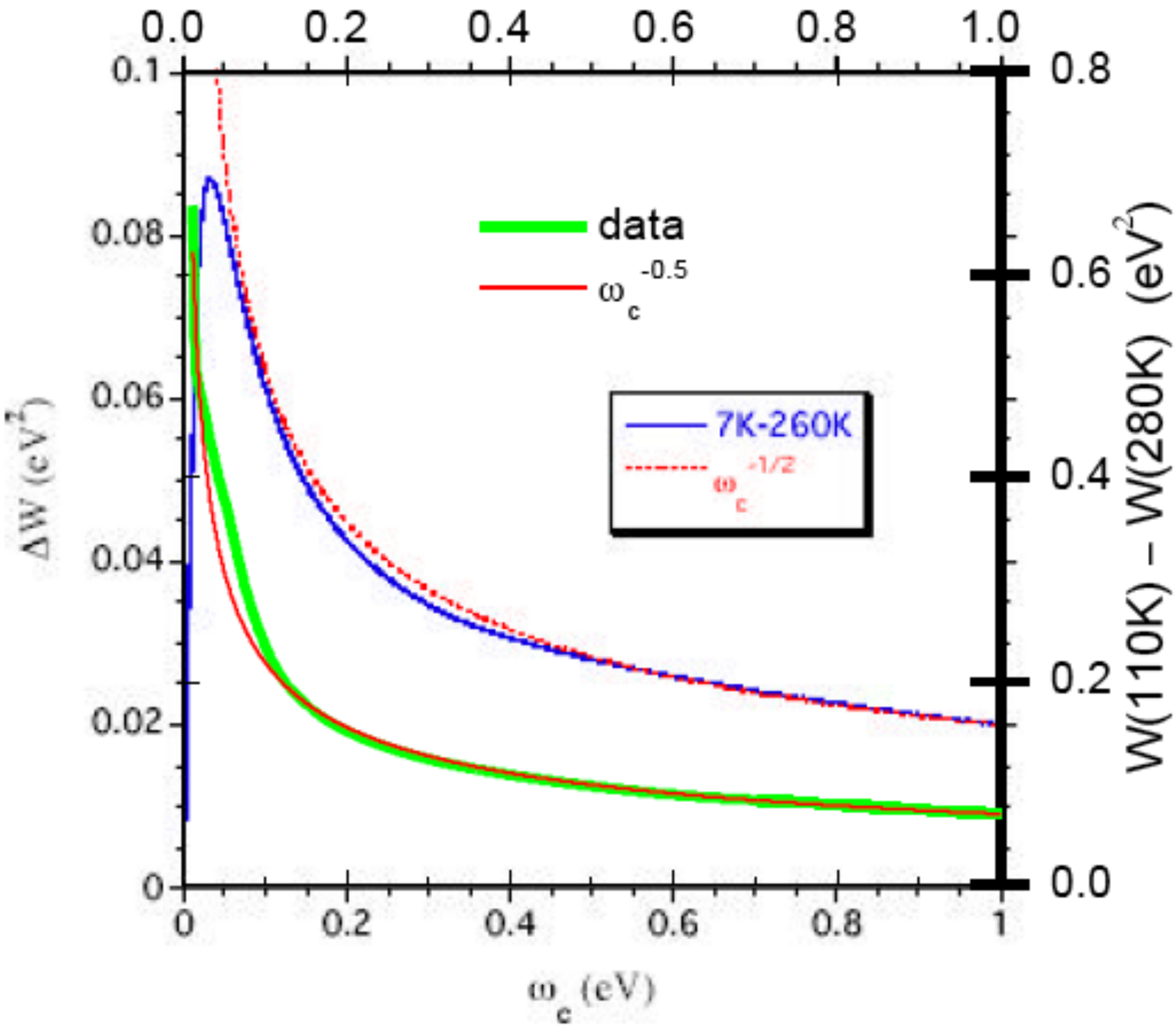


r_i=26.5
7K-260K
$\omega_c^{-1/2}$
ΔW (eV^2)
ω_c (eV)
1
0.1
0.01
0.001
0.001
0.01
0.1
1
log(W)
log(ω_c)
0
-1
-2
-3
-3
-2
-1
0

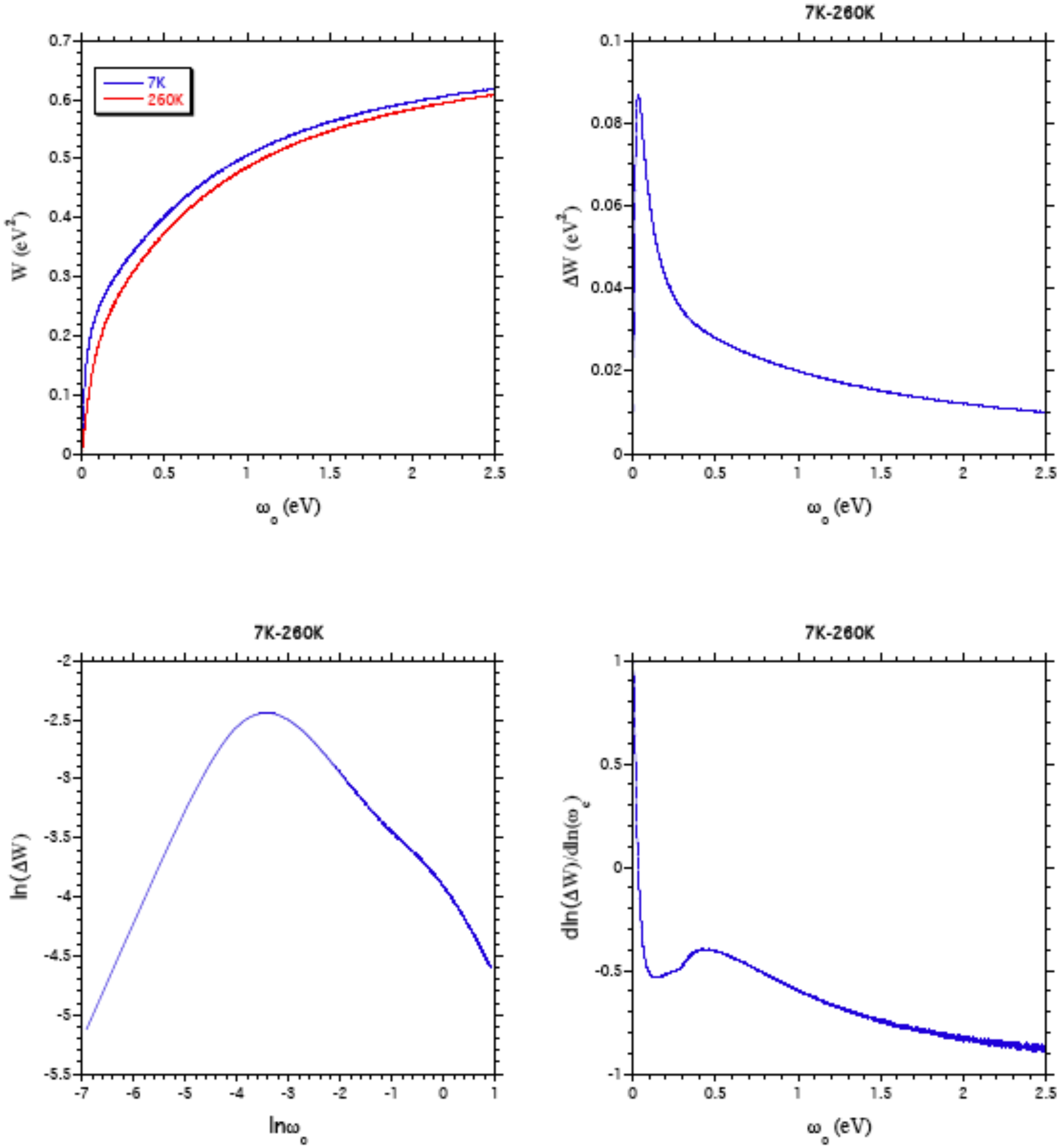
Fig. 1



T^2 (10^4 K^2)
0 1 2 3 4 5 6 7
0.51 0.5 0.49 0.48 0.47 0.46 0.45
4.08 4.00 3.92 3.84 3.76 3.68 3.60
W (eV^2) @ 1eV
8$\hbar^2$W (eV^2)
0 1 10^4 2 10^4 3 10^4 4 10^4 5 10^4 6 10^4 7 10^4
T^2 (K^2)



0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.08
0.06
0.04
0.02
0
data
ωc-0.5
7K-260K
ωc-1/2
ΔW (eV2)
W(110K) – W(280K) (eV2)
0.8
0.6
0.4
0.2
0.0
0
0.2
0.4
0.6
0.8
1
ωc (eV)

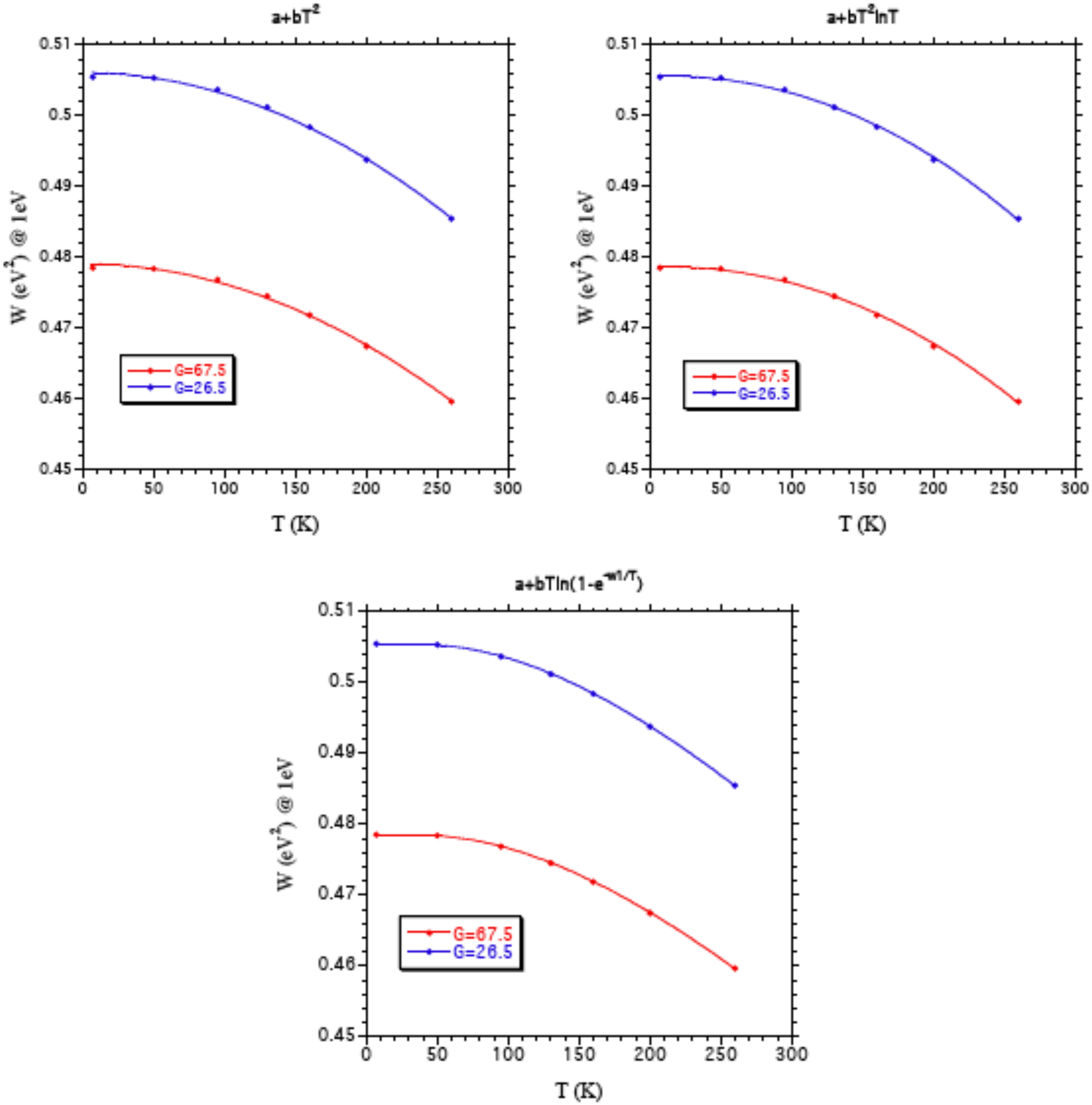

7K
260K
W (eV^2)
ω_o (eV)
7K-260K
ΔW (eV^2)
ω_o (eV)
7K-260K
ln(ΔW)
lnω_o
7K-260K
dln(ΔW)/dln(ω_c)
ω_o (eV)

$$W(\Omega,T) = const - \int_{\Omega}^{\infty} Re\sigma(\omega,T)d\omega$$

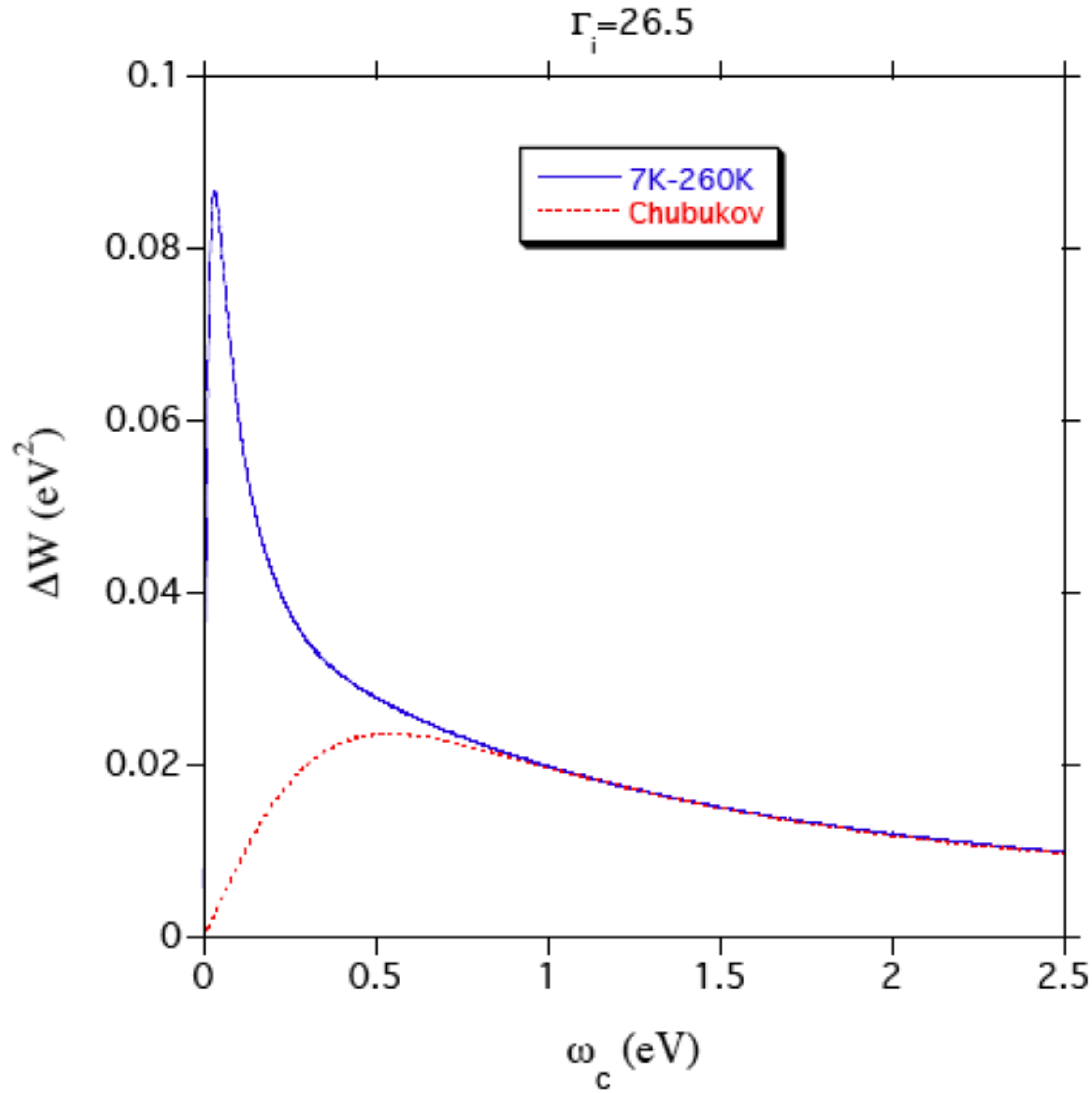
$$1/\tau_{high} = 2\Gamma_i + \frac{\Gamma}{\omega_2 - \omega_1}(4T \ln \frac{\sinh \frac{\omega_2}{2T}}{\sinh \frac{\omega_1}{2T}} - \frac{\omega_2^2 - \omega_1^2}{\omega})$$

$$\tau_{high} = \tau_0 + (4T\Gamma\tau_0^2/\omega_2) * \log(1 - e^{-\omega_1/T})$$

$$\delta W(\Omega,T) = \frac{\omega_{pl}^2}{4\pi}\frac{2\omega_1}{\omega_2}T^* \log(1 - e^{-1/T^*})\frac{\Omega^*}{1 + (\Omega^*)^2}$$



a+bT²
a+bT²lnT
a+bTln(1-e^-w1/T)
W (eV²) @ 1eV
T (K)
G=67.5
G=26.5
0.51
0.5
0.49
0.48
0.47
0.46
0.45
0
50
100
150
200
250
300



Γi=26.5
7K-260K
Chubukov
ΔW (eV2)
ωc (eV)
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5

CONCLUSIONS

Now, we implicitly assumed a quadratic dispersion, so E_K is technically conserved in our case

SO, THE MORAL OF THIS STORY IS

1. E_K and $E_{kinetic}$ are in general ***not*** the same thing

2. An analysis of the cut-off behavior of E_K is necessary to determine whether there is a residual "violation" of $\Delta E_K=0$